Lab 4 Tuesday September 18

Trigonometry

1. What is `Sin[0]`? Enter it into Mathematica and see what you get. Plug in several numbers getting close to zero; what happens?
2. Plot a graph using the code `Plot[Sin[x],{x,-1,1}]`. Does this match the numbers you got in the previous step?
3. Enter `Limit[Sin[x],x -> 0]`. What do you get?
4. Now define a function `h[x_]:=Sin[1/x]`. What is its value at 0? Enter `h[0]` into Mathematica and see what you get.
5. Plug in about ten numbers getting close to 0. What happens?
6. Plot a graph of `h[x]` with domain $[-1, 1]$. What happens near zero? Try using smaller domains like $[-.1, .1]$ or $[-.01, .01]$.
7. Use the `Limit` command to compute the limit at 0. What do you think this means?
8. Define `j[x_]:=x * Sin[1/x]`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
9. Plot a graph of `j[x]` with domain $[-1, 1]$. What happens near zero? Plot more graphs with smaller domains.
10. Use the `Limit` command to compute the limit at 0.
11. Define `k[x_]:=Sin[x]/x`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
12. Plot a graph of `k[x]` with domain $[-\pi, \pi]$. What happens near zero? Plot more graphs with smaller domains. Also plot some larger domains. Do these pictures match your expectations?
13. Use the `Limit` command to compute the limit at 0.

The Squeeze Theorem

A principle called the "Squeeze Theorem" or the "Two Policemen Theorem" allows us to compute the limit of a function we don't like by "trapping" or "squeezing" it between two functions which do. In the rest of this lab we'll visualize a few examples. We'll discuss this in more detail after the test.

1. Earlier in this lab, we considered the function `x*Sin[1/x]` and its limit at zero. Now we want to generate better understanding of its behavior there.
   (a) Plot the function $x\sin(1/x)$ with the code `Plot[x * Sin[1/x],{x,-1,1}]`.
   (b) Now plot $x\sin(1/x)$ on the same graph as $|x|$ and $-|x|$, with the code `Plot[{x * Sin[1/x], Abs[x],-Abs[x]},{x,-1,1}]`
   (c) Shrink the domain around the point $x = 0$ to see what happens.
   (d) What are the limits of $|x|$ and $-|x|$ at 0? What does this tell us about the limit of $x\sin(1/x)$?

2. We also looked at the important limit `Sin[x]/x`.
   (a) As we did before, plot a graph of `Sin[x]/x`, along with the functions `1` and `Cos[x]`. (Again, for a bonus, color-code your graph).
   (b) Shrink the domain around $x = 0$ to see what happens. What are the limits of 1 and of $\cos(x)$ at 0?

3. Now we'll practice finding these bounds.
   (a) Using the fact that $-1 \leq \sin(a) \leq 1$ for any a, find upper and lower bounds for $\cos\left(\frac{32+x}{x+1}\right)$. You should be able to find a number that's always bigger, and a number that's always smaller.
   (b) Use your answer to find bounds for $\cos^2\left(\frac{32+x}{x+1}\right)$. Again, you should find a number that's always bigger and a number that's always smaller.
   (c) Now find bounds for $(x+1)\cos^2\left(\frac{32+x}{x+1}\right)$. You should have a *function* that's always bigger and a function that's always smaller.
   (d) Plot all three functions from the previous part with domain $[-2, 0]$. Is your upper bound actually always above the function? Is your lower bound always below? Make sure the bounds don't cross *through* the function. If they do, can you fix this?
   (e) What does this picture suggest about $\lim x \to -1(x+1)\cos^2\left(\frac{32+x}{x+1}\right)$?

4. Use the same process for $(x+1)^2\cos^2\left(\frac{32+x}{x+1}\right)$. What's different here?