Math 114 Fall 2018
Calculus I Practice Homework 3.5 Solutions
Do not turn in

1. Prove that $\lim_{x\to 5} \frac{x-2}{x-5} = \pm\infty$.

   Solution: Let $N > 0$ and set $\delta \leq \underline{2, 1/N}$. Then if $0 < |x-5| < \delta$, we have

$$\begin{aligned} \left|\frac{x-2}{x-5}\right| &= \frac{|x-2|}{|x-5|} > \frac{|x-2|}{\delta} \\ &= \frac{|x-5+3|}{\delta} \geq \frac{3-|x-5|}{\delta} \\ &> \frac{3-\delta}{\delta} \geq \frac{1}{\delta} \geq \frac{1}{1/N} = N. \end{aligned}$$

   Thus $\lim_{x\to 5} \frac{x-2}{x-5} = \pm\infty$.

2. Prove that $\lim_{x\to -3} \frac{-1}{(x+3)^4} = -\infty$.

   Solution: Let $N > 0$ and set $\delta = \underline{1/\sqrt[4]{N}}$. Then if $|x+3| < \delta$, we have

$$\begin{aligned} \frac{1}{(x+3)^4} &> \frac{1}{\delta^4} = \frac{1}{1/N} = N \\ \frac{-1}{(x+3)^4} &< \frac{-1}{\delta^4} = \frac{-1}{1/N} = -N. \end{aligned}$$

   Thus $\lim_{x\to -3} (x+3)^4 = -\infty$.

3. Prove that $\lim_{x\to 4} \frac{3}{(x-4)^2} = +\infty$.

   Solution: Let $N > 0$ and set $\delta \leq \underline{\sqrt{3/N}}$. If $0 < |x-4| < \delta$, then

$$\left|\frac{3}{(x-4)^2}\right| = \frac{3}{|x-4|^2} > \frac{3}{\delta^2} \geq \frac{3}{(\sqrt{3/N})^2} - N.$$

4. Explicitly naming the rule used in each step, calculate $\lim\limits_{x\to 0} x^2 - 3x + 5$

Solution:

$$\begin{aligned}
\lim_{x\to 0} x^2 - 3x + 5 &= \lim_{x\to 0} x^2 - \lim_{x\to 0} 3x + \lim_{x\to 0} 5 && \text{additivity}\\
&= \left(\lim_{x\to 0} x\right)^2 - 3\lim_{x\to 0} x + \lim_{x\to 0} 5 && \text{exponents, scalars}\\
&= (0)^2 - 3\cdot 0 + 5 && \text{identity, constants}\\
&= 0 - 0 + 5 = 5.
\end{aligned}$$

5. Explicitly naming the rule used in each step, calculate $\lim_{x\to 4} \sqrt{x} + \sqrt[3]{4+x}$

Solution:

$$\begin{aligned}
\lim_{x\to 4} \sqrt{x} + \sqrt[3]{4+x} &= \lim_{x\to 4} \sqrt{x} + \lim_{x\to 4} \sqrt[3]{4+x} && \text{additivity}\\
&= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4+x} && \text{exponents}\\
&= \sqrt{\lim_{x\to 4} x} + \sqrt[3]{\lim_{x\to 4} 4 + \lim_{x\to 4} x} && \text{additivity}\\
&= \sqrt{4} + \sqrt[3]{4+4} && \text{identity, constants}\\
&= 2 + 2 = 4.
\end{aligned}$$

6. Explicitly naming the rule used in each step, calculate $\lim_{x\to 2} f(x)$ where

$$f(x) = \begin{cases} x+1 & x < 2\\ x^2 - 1 & x > 2 \end{cases}$$

Solution:

We need to calculate the two one-sided limits. We have

$$\begin{aligned}
\lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} x + 1 && \text{Almost Identical Functions}\\
&= \lim_{x\to 2^-} x + \lim_{x\to 2^-} 1 && \text{Additivity}\\
&= \lim_{x\to 2^-} x + 1 && \text{Constants}\\
&= 2 + 1 = 3 && \text{Identity}\\
\lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} x^2 - 1 && \text{Almost Identical Functions}\\
&= \lim_{x\to 2^+} x^2 - \lim_{x\to 2^+} 1 && \text{Additivity}\\
&= \left(\lim_{x\to 2^+} x\right)^2 - \lim_{x\to 2^+} && \text{Exponents}\\
&= (2)^2 - \lim_{x\to 2^+} 1 && \text{Identity}\\
&= 4 - 1 = 3 && \text{Constants}
\end{aligned}$$

We see that $\lim_{x\to 2^+} f(x) = 3 = \lim_{x\to 2^-} f(x)$, so by agreement of the left and right limits, $\lim_{x\to 2} f(x) = 3$.