

Math 114 Fall 2018
Calculus I Practice Homework 7.5 Solutions
Do not turn in

1. Suppose $f'(x) = \frac{1}{2}f(x)$ and $f(1) = 3$. Use Euler's method to approximate $f(4)$ using three steps.

Solution: We have

$$\begin{aligned}f(2) &\approx f'(1)(2-1) + f(1) = 3/2 \cdot 1 + 3 = 4.5 \\f(3) &\approx f'(2)(3-2) + f(2) \approx 9/4 + 9/2 = 27/4 \\f(4) &\approx f'(3)(4-3) + f(3) \approx 27/8 + 27/4 = 81/8.\end{aligned}$$

2. Suppose $f'(x) = 4 - \frac{f(x)}{x}$, and $f(2) = 2$. Use Euler's method and four steps to approximate $f(4)$.

Solution:

$$\begin{aligned}f(5/2) &\approx f'(2)(.5) + f(2) = 3/2 + 2 = 7/2 \\f(3) &\approx f'(5/2)(1/2) + f(5/2) \\&\approx \left(4 - \frac{7/2}{5/2}\right) / 2 + 7/2 = 13/10 + 7/2 = 24/5 \\f(7/2) &\approx f'(24/5)(1/2) + f(3) \\&\approx \left(4 - \frac{24/5}{3}\right) / 2 + 24/5 = 6/5 + 24/5 = 6 \\f(4) &\approx f'(7/2)(1/2) + f(7/2) \\&\approx \left(4 - \frac{6}{7/2}\right) / 2 + 6 = 8/7 + 6 = 50/7.\end{aligned}$$

3. Suppose $f'(x) = xf(x)$, and $f(0) = 3$. Use four steps to estimate $f(4)$.

Solution:

$$\begin{aligned}f(1) &\approx f'(0)(1-0) + f(0) = 0 \cdot 1 + 3 = 3 \\f(2) &\approx f'(1)(2-1) + f(1) \approx (1 \cdot 3) \cdot 1 + 3 = 6 \\f(3) &\approx f'(2)(3-2) + f(2) \approx (2 \cdot 6) \cdot 1 + 6 = 18 \\f(4) &\approx f'(3)(4-3) + f(3) \approx (3 \cdot 18) \cdot 1 + 18 = 72.\end{aligned}$$

4. Suppose $f'(x) = e^x$ and $f(0) = 1$. Use four steps to estimate $f(4)$.

Solution:

$$\begin{aligned}f(1) &\approx f'(0)(1 - 0) + f(0) = 1 \cdot 1 + 1 = 2 \\f(2) &\approx f'(1)(2 - 1) + f(1) \approx e^1 + 2 \\f(3) &\approx f'(2)(3 - 2) + f(2) \approx e^2 + e + 2 \\f(4) &\approx f'(3)(4 - 3) + f(3) \approx e^3 + e^2 + e + 2.\end{aligned}$$

5. Suppose $f'(x) = f(x)$ and $f(0) = 1$. Use four steps to estimate $f(4)$.

Solution:

$$\begin{aligned}f(1) &\approx f'(0)(1 - 0) + f(0) = 1 \cdot 1 + 1 = 2 \\f(2) &\approx f'(1)(2 - 1) + f(1) \approx 2 \cdot 1 + 2 = 4 \\f(3) &\approx f'(2)(3 - 2) + f(2) \approx 4 \cdot 1 + 4 = 8 \\f(4) &\approx f'(3)(4 - 3) + f(3) \approx 8 \cdot 1 + 8 = 16\end{aligned}$$

6. Suppose $f'(x) = \sin(f(x)) - x$ and $f(0) = 2$. Use three steps to estimate $f(\pi)$.

Solution:

$$\begin{aligned}f(\pi/3) &\approx f'(0)(\pi/3 - 0) + f(0) = (\sin(2) - 0)(\pi/3 - 0) + 2 \approx 2.95 \\f(2\pi/3) &\approx f'(\pi/3)(2\pi/3 - \pi/3) + f(\pi/3) \approx (2.95 - \pi/3)(\pi/3) + 2.95 \approx 4.94 \\f(\pi) &\approx f'(2\pi/3)(\pi - 2\pi/3) + f(2\pi/3) \approx (4.94 - 2\pi/3)(\pi/3) + 4.94 \approx 7.92.\end{aligned}$$

7. Suppose $f'(x) = f(x) - x$ and $f(1) = 3$. Use one step to estimate $f(2)$.

Now use two steps to estimate $f(2)$. Now use four steps to estimate $f(2)$. What happens?

Solution: One step:

$$f(2) \approx f'(1)(2 - 1) + f(1) = (3 - 1)(1) + 3 = 5$$

Two steps:

$$\begin{aligned}f(3/2) &\approx f'(1)(3/2 - 1) + f(1) = (3 - 1)(1/2) + 3 = 4 \\f(2) &\approx f'(3/2)(2 - 3/2) + f(3/2) \approx (4 - 3/2)(1/2) + 4 = 21/4 = 5.25\end{aligned}$$

Four steps:

$$\begin{aligned}f(5/4) &\approx f'(1)(5/4 - 1) + f(1) = (3 - 1)(1/4) + 3 = 7/2 \\f(3/2) &\approx f'(5/4)(3/2 - 5/4) + f(5/4) \approx (7/2 - 5/4)(1/4) + 7/2 = 65/16 \approx \\f(7/4) &\approx f'(3/2)(7/4 - 3/2) + f(3/2) \approx \left(\frac{65}{16} - \frac{3}{2}\right)(1/4) + \frac{65}{16} = \frac{301}{64} \approx 4.0625 \\f(2) &\approx f'(7/4)(2 - 7/4) + f(7/4) \approx \left(\frac{301}{64} - \frac{7}{4}\right)(1/4) + \frac{301}{64} = \frac{1393}{256} \approx 5.44141.\end{aligned}$$

Each successive answer takes more work, but the answers are drifting upwards, suggesting all of them are underestimates converging to some higher answer.

8. Stewart 2.7.3
9. Stewart 2.7.5
10. Stewart 2.7.15
11. Stewart 2.7.19
12. Stewart 2.7.25
13. (★) Stewart 2.7.37