

**Problem 1.** (a) Use the definition of limit to prove that  $\lim_{x \rightarrow 2} \frac{1}{x+3} = \frac{1}{5}$ .

(b) Use the definition of limit to prove that  $\lim_{x \rightarrow 1} \frac{1}{(x-1)^2} = +\infty$ .

**Problem 2.** (a) Use the Squeeze Theorem to show that  $\lim_{x \rightarrow 5} (x - 5) \sin\left(\frac{x^2 + 1}{x - 5}\right) = 0$ .

(b) Compute  $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

(c) Compute  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{\sin^2(x)}$

**Problem 3.** (a) **Directly from the definition**, compute  $f'(1)$  where  $f(x) = \sqrt{x+3}$ .

(b) Compute  $g'(x)$  where  $g(x) = \ln \left| \frac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2 + 1}} \right|$ .

(c) Find a tangent line to the function  $f(x) = \frac{e^x}{x}$  at the point given by  $x = 2$ .

**Problem 4.** (a) Let  $g(x) = \sqrt[5]{x^9 + x^7 + x + 1}$ . Find  $(g^{-1})'(1)$ .

(b) Write a tangent line to the curve  $y^2 = x^{x \cos(x)}$  at the point  $(\pi/2, -1)$ .

(c) Find  $y'$  if  $e^y + \ln(y) = x^2 + 1$ .

**Problem 5.** (a) A cone with height  $h$  and base radius  $r$  has volume  $\frac{1}{3}\pi r^2 h$ . Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at  $\frac{1}{9\pi}$  meters per minute, what volume of water leaks out every minute?

(b) Use two iterations of Newton's method, starting at 0, to estimate the root of  $e^x - 3x$ .

(c) Find all the critical points of  $g(x) = \ln(x^3 + 9x^2 + 27x)$ .

**Problem 6.** (a) If  $f(x) = \sqrt{x} + \tan(\pi x)$ , use a linear approximation centered at 4 to estimate  $f(4.1)$ .

(b) If  $g(x) = \cos(x)$ , use a quadratic approximation centered at 0 to estimate  $g(.1)$ .

(c) Let  $g'(x) = g(x) + 3x$ , and  $g(2) = 4$ . Use two steps of Euler's method to estimate  $g(4)$ . Is this an overestimate or an underestimate?

**Problem 7.** (a) Find the absolute extrema of  $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$  on  $[0, 5]$ .

(b) Ten miles from home you remember that you left the water running, which is costing you 90 cents an hour. Driving home at speed  $s$  miles per hour costs you  $4(s/10)$  cents per mile. At what speed should you drive to minimize the total cost of gas and water?

(c) Classify the relative extrema of  $h(x) = \sqrt[3]{x}(x + 4)$

**Problem 8.** (a) Find all the critical points of  $g(x) = \frac{x^2 - 8}{x + 3}$

(b) If  $-1 \leq f'(x) \leq 3$  and  $f(0) = 0$ , what can you say about  $f(4)$ ? Assume  $f$  is continuous and differentiable.

(c) Prove that  $x^2 - (e^2 + 1)\ln(x)$  has exactly two real roots.

**Problem 9.** Let  $j(x) = x^4 - 14x^2 + 24x + 6$ . We can compute  $j'(x) = 4(x + 3)(x - 1)(x - 2)$  and  $j''(x) = 4(3x^2 - 7)$ . Sketch a graph of  $j$ .

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

**Problem 10.** Let  $g(x) = \arctan(x^2 + x)$ . We can compute that  $g'(x) = \frac{2x+1}{1+(x^2+x)^2}$  and

$$g''(x) = \frac{-6x^4 - 12x^3 - 8x^2 - 2x + 2}{(1 + (x^2 + x)^2)^2}.$$

Sketch a graph of  $g$ .

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.