Problem 1. (a) Use the definition of limit to prove that $\lim_{x\to 2} \frac{1}{x+3} = \frac{1}{5}$.

(b) Use the definition of limit to prove that $\lim_{x\to 1} \frac{1}{(x-1)^2} = +\infty$.

Problem 2. (a) Use the Squeeze Theorem to show that $\lim_{x\to 5} (x-5) \sin\left(\frac{x^2+1}{x-5}\right) = 0$.

(b) Compute
$$\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$$

(c) Compute
$$\lim_{x\to 0} \frac{\sin(x^2)}{\sin^2(x)}$$

Problem 3. (a) Directly from the definition, compute f'(1) where $f(x) = \sqrt{x+3}$.

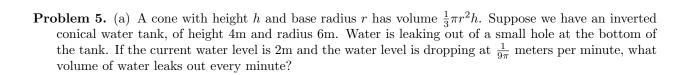
(b) Compute
$$g'(x)$$
 where $g(x) = \ln \left| \frac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2 + 1}} \right|$.

(c) Find a tangent line to the function $f(x) = \frac{e^x}{x}$ at the point given by x = 2.

Problem 4. (a) Let $g(x) = \sqrt[5]{x^9 + x^7 + x + 1}$. Find $(g^{-1})'(1)$.

(b) Write a tangent line to the curve $y^2 = x^{x \cos(x)}$ at the point $(\pi/2, -1)$.

(c) Find y' if $e^y + \ln(y) = x^2 + 1$.



(b) Use two iterations of Newton's method, starting at 0, to estimate the root of $e^x - 3x$.

(c) Find all the critical points of $g(x) = \ln(x^3 + 9x^2 + 27x)$.



(b) If $g(x) = \cos(x)$, use a quadratic approximation centered at 0 to estimate g(.1).

(c) Let g'(x) = g(x) + 3x, and g(2) = 4. Use two steps of Euler's method to estimate g(4). Is this an overestimate or an underestimate?

Problem 7. (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on [0, 5].

(b) Ten miles from home you remember that you left the water running, which is costing you 90 cents an hour. Driving home at speed s miles per hour costs you 4(s/10) cents per mile. At what speed should you drive to minimize the total cost of gas and water?

(c) Classify the relative extrema of $h(x) = \sqrt[3]{x}(x+4)$

Problem 8. (a) Find all the critical points of $g(x) = \frac{x^2 - 8}{x + 3}$

(b) If $-1 \le f'(x) \le 3$ and f(0) = 0, what can you say about f(4)? Assume f is continuous and differentiable.

(c) Prove that $x^2 - (e^2 + 1) \ln(x)$ has exactly two real roots.

Problem 9. Let $j(x) = x^4 - 14x^2 + 24x + 6$. We can compute j'(x) = 4(x+3)(x-1)(x-2) and $j''(x) = 4(3x^2 - 7)$. Sketch a graph of j.

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

Problem 10. Let $g(x) = \arctan(x^2 + x)$. We can compute that $g'(x) = \frac{2x+1}{1+(x^2+x)^2}$ and

$$g''(x) = \frac{-6x^4 - 12x^3 - 8x^2 - 2x + 2}{(1 + (x^2 + x)^2)^2}.$$

Sketch a graph of g.

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.