Math 310 Fall 2018
Real Analysis HW 1
Due Friday, September 7

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem. Submit this problem on a separate, detached sheet of paper.

$\star$ **Redo Problem:** Let F be an ordered field, and $x, y, z \in F$. Prove that if $x < 0$ and $y < z$ then $xy > xz$.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Let F be a field and let $x_1, \ldots, x_n \in F$. Prove that the value of the expression $x_1 + x_2 + \cdots + x_n$ does not depend on the order of the terms, and any reordering will give the same value. (Hint: Use induction on the number of terms, and use the commutative property as your base case).

2. Prove that if F is a field and $x \in F$, then $0x = 0$.

3. Let F be an ordered field, and let $x, y \in F$ with $0 < x < y$. Prove that $x^2 < y^2$.

4. Let F be an ordered field and let $x, y, z, w \in F$. If $x \leq y, z \leq w$, prove that $x+z \leq y+w$. (Note: you may need to consider multiple cases here).

5. Prove that $\mathbb{C}$ cannot be an ordered field. (Hint: is $i > 0$, $i = 0$, or $i < 0$?)

6. For $a, b \in \mathbb{R}$, show that

$$\max\{a, b\} = \frac{a + b + |a - b|}{2}$$
$$\min\{a, b\} = \frac{a + b - |a - b|}{2}$$

7. Let F be an ordered field and $S \subset F$. Let y be a least upper bound of S, and let $x < y$. Prove that there is an $s \in S$ such that $x < s$.

8. Does the empty set have a least upper bound in $\mathbb{R}$? If yes, find it and prove it is a least upper bound. If no, prove that no least upper bound exists.

9. Let S be an ordered set and $A \subset S$. Suppose b is an upper bound for A, and that $b \in A$. Prove that $b = \sup A$.