

Math 310 Fall 2018
Real Analysis HW 10
Due Monday, November 19

For all of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Prove that $\int_0^1 x \, dx = 1/2$ directly from the definition of Riemann integral. (You may assume that the integral exists if you wish).
2. Let $c \in (a, b)$, and let $f : [a, b] \rightarrow \mathbb{R}$ be defined by $f(c) = 1$ and $f(x) = 0$ if $x \neq c$. Prove that $\int_a^b f(x) \, dx = 0$.
3. Let $f : [a, b] \rightarrow \mathbb{R}$ such that $\int_a^b f(x) \, dx$ exists. Prove that $\int_{a+c}^{b+c} f(x-c) \, dx$ exists, and is equal to $\int_a^b f(x) \, dx$.
4. If $f : [a, b] \rightarrow \mathbb{R}$ is an integrable function and $c \in \mathbb{R}$, prove that $cf(x)$ is integrable and

$$\int_a^b cf(x) \, dx = c \int_a^b f(x) \, dx.$$