Math 310 Fall 2018
Real Analysis HW 10
Due Monday, December 3

For all these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Suppose $f : [a, b] \to \mathbb{R}$ is integrable. Show that f is bounded.

2. Prove directly from the definition that $f : (2, 4) \to \mathbb{R}$ defined by $f(x) = \frac{1}{x}$ is uniformly continuous.

3. Let $f : (0, 1) \to \mathbb{R}$ be defined by $f(x) = \sin(1/x)$. Show that f is not uniformly continuous.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Show that f is not uniformly continuous.

5. Let $f : [a, b] \to \mathbb{R}$ be continuous, such that $f(x) \geq 0$ for every $x \in [a, b]$. Suppose $c \in [a, b]$ with $f(c) > 0$. Prove that $\int_a^b f(x)\,dx > 0$.

   (Note: this result is not true if f is not continuous. See e.g. HW 10 number 2.)

6. Use Fact 5.19 to prove that if $f : [a, b] \to \mathbb{R}$ is increasing, then $\int_a^b f(x)\,dx$ exists.