Math 310 Fall 2018
Real Analysis HW 2
Due Friday, September 14

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem. Submit this problem on a separate, detached sheet of paper.

$\star$ **Redo Problem:** Let S_1, S_2 be non-empty subsets of $\mathbb{R}$ that are bounded above. Let $S = \{s_1 + s_2 : s_1 \in S_1, s_2 \in S_2\}$. Prove that $\sup S = \sup S_1 + \sup S_2$.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Prove that 1 is the least upper bound of $\{1 - \frac{1}{n} : n \in \mathbb{N}\}$.

2. Let S be a subset of an ordered field. We say that y is a *lower bound* for S if $y \leq S$ for all $s \in S$. We say that y is a *greatest lower bound* for S, and write $y = \inf(S)$ if y is a lower bound for S, and if x is also a lower bound for S then $y \geq x$.

   Prove that if S has a greatest lower bound then that lower bound is unique.

3. Let $S \subset \mathbb{R}$ be non-empty and bounded below. Prove that S has a greatest lower bound.

4. Let $S = \{\frac{1}{n^2} : n \in \mathbb{N}\}$. Find the greatest lower bound for S, and prove it is the greatest lower bound.

5. Prove that for any real number $\epsilon > 0$, there is a $n \in \mathbb{N}$ such that $1/n < \epsilon$.

6. Prove that if $x < y$, then $x^3 < y^3$.

7. Prove that if x is a real number, there is a unique real number y such that $y^3 = x$.

8. Let $a < b$ be real numbers. Prove there is a rational number r with $a < r < b$.

9. Prove that $\mathbb{R}^2$ with the metric $d((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|$ is a metric space.