# Math 310 Fall 2018
# Real Analysis HW 4
# Due Friday, September 28

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem. Submit this problem on a separate, detached sheet of paper.

$\star$ **Redo Problem:** Let $E$ be a metric space under the discrete metric. Prove that $\lim_{n\to\infty} x_n = x$ converges if and only if there is some $N \in \mathbb{N}$ such that $x_n = x$ for every $n > N$.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. If you didn't already, finish problem 7 from last homework: show that the set $V = \{(x_1, x_2) : x_1 x_2 = 1, x_1 > 0\}$ is closed in $\mathbb{R}^2$ with one of the Euclidean, sup, or sum metric.

2. Let $(E, d)$ be a metric space, and let $S \subset E$. Prove that $S$ is bounded if and only if the set $\{d(x, y) : x, y \in S\} \subset \mathbb{R}$ is bounded above.

3. Let $E = \mathbb{R}^2$ and consider the sequence $\left(\frac{1}{n}, 1 - \frac{1}{n}\right) = (1, 0), (1/2, 1/2), (1/3, 2/3), \ldots$. Prove that this sequence converges in one of the sup, sum, or Euclidean metric.

4. Let $E = \mathbb{R}$ and let $x_n = n$. Prove that $(x_n)$ does not converge in the regular absolute value metric.

5. Let $(E, d)$ be a metric space, and let $V$ be a closed subset of $E$. Prove that $\overline{V} = V$.

6. Let $(E, d)$ be a metric space, and $x \in E$. Is the closure of $B_r(x)$ always equal to the closed ball $\overline{B}_r(x)$? Either prove it, or find a counterexample.

7. If $(E, d)$ is a metric space and $U \subset E$, prove the interior of $U$ is the set of all points $x \in U$ such that some open ball containing $x$ is also a subset of $U$.

8. Consider the metric space $\mathbb{R}$ under the usual metric. Find the interior of $[0, 1]$ and prove your answer.