

Math 310 Fall 2018  
Real Analysis HW 5  
Due Wednesday, October 3

No redo problem this week.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. If  $\lim_{n \rightarrow \infty} a_n = a$  and  $\lim_{n \rightarrow \infty} b_n = b$ , and  $a_n \leq b_n$  for all  $n$ , prove that  $a \leq b$ .
2. Let  $a_n, b_n \in \mathbb{R}$  such that  $\lim_{n \rightarrow \infty} a_n = a$  and  $\lim_{n \rightarrow \infty} b_n = b$ . Prove that  $\lim_{n \rightarrow \infty} (a_n b_n) = ab$ .
3. Let  $S \subset \mathbb{R}$  be nonempty and bounded above. Prove there is a monotone sequence  $(x_n)$  such that  $x_n \in S$  and  $\lim_{n \rightarrow \infty} x_n = \sup S$ .