Math 310 Fall 2018
Real Analysis HW 7
Due Friday, October 26

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem. Submit this problem on a separate, detached sheet of paper.

⋆ **Redo Problem:** Let (E, d) be a metric space, and suppose $\lim_{n\to\infty} x_n = x$. Prove that x is an accumulation point of the sequence (x_n).

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Let $S = \{\frac{1}{n} + \frac{1}{m} : n, m \in \mathbb{N}\}$. Find (with proof) all cluster points of S.
2. Let (E, d) be a metric space, and let $V \subset E$. Prove that V is closed if and only if it contains all its accumulation points.
3. Let $S \subset \mathbb{R}$ be a set that is non-empty and bounded above, but has no greatest element. Prove $\sup S$ is a cluster point for S.
4. Let $(x_n) = 0, 1, 2, 0, 1, 2, \ldots$. Prove that $0, 1, 2$ are all accumulation points of (x_n).
5. Let $x_n = \begin{cases} 3/n & n \text{ odd} \\ 2 + 2/n & n \text{ even} \end{cases}$

   Find all accumulation points of x_n and prove they are accumulation points.
6. If (x_n) is a bounded sequence of real numbers, prove that $\liminf x_n$ is an accumulation point.
7. (a) Find a subset of $[0, 1]$ that doesn't have an accumulation point. Why does this not contradict the fact that $[0, 1]$ is compact?

   (b) Find an infinite subset of $\mathbb{R}$ that doesn't have an accumulation point.
8. Let (E, d) be a metric space, and let $V_1, \ldots, V_n$ be a finite collection of (topologically) compact subsets. Prove that $V_1 \cup \cdots \cup V_n$ is (topologically) compact.