

Lab 4**Thursday February 22****Trigonometry**

1. Define a function $h[x_] := \text{Sin}[1/x]$. What is $h[0]$? Plug it into Mathematica and see what answer you get.
2. Plug in about ten numbers getting close to 0. What happens?
3. Plot a graph of $h[x]$ with domain $[-1, 1]$. What happens near zero? Try using smaller domains like $[-.1, .1]$ or $[-.01, .01]$.
4. Use the `Limit` command to compute the limit at 0. What do you think this means?
5. Define $j[x_] := x * \text{Sin}[1/x]$. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
6. Plot a graph of $j[x]$ with domain $[-1, 1]$. What happens near zero? Plot more graphs with smaller domains.
7. Use the `Limit` command to compute the limit at 0.
8. Define $k[x_] := \text{Sin}[x]/x$. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
9. Plot a graph of $k[x]$ with domain $[-\pi, \pi]$. What happens near zero? Plot more graphs with smaller domains. Also plot some larger domains. Do these pictures match your expectations?
10. Use the `Limit` command to compute the limit at 0.

The Squeeze Theorem

A principle called the “Squeeze Theorem” or the “Two Policemen Theorem” allows us to compute the limit of a function we don’t like by “trapping” or “squeezing” it between two functions which do. In the rest of this lab we’ll visualize a few examples.

1. Earlier in this lab, we considered the function $x \cdot \sin[1/x]$ and its limit at zero. Now we want to generate better understanding of its behavior there.
 - (a) Plot $x \cdot \sin[1/x]$ on the same graph as $\text{Abs}[x]$ and $-\text{Abs}[x]$. (Bonus: color-code the graph so $x \cdot \sin[1/x]$ is one color and the other two graphs are a different color, using the `PlotStyle` command).
 - (b) Shrink the domain around the point $x = 0$ to see what happens.
 - (c) What are the limits of $|x|$ and $-|x|$ at 0? What does this tell us about the limit of $x \sin(1/x)$?
2. We also looked at the important limit $\sin[x]/x$.
 - (a) As we did before, plot a graph of $\sin[x]/x$, along with the functions 1 and $\cos[x]$. (Again, for a bonus, color-code your graph).
 - (b) Shrink the domain around $x = 0$ to see what happens. What are the limits of 1 and of $\cos(x)$ at 0?
3. Now we’ll practice finding these bounds.
 - (a) Using the fact that $-1 \leq \sin(a) \leq 1$, find upper and lower bounds for $\sin\left(\frac{74}{x-3}\right)$.
 - (b) Use your answer to find bounds for $\sin^2\left(\frac{74}{x-3}\right)$ and then for $1 + \sin^2\left(\frac{74}{x-3}\right)$. Plot all three functions on the domain $\{x, 2, 4\}$.
 - (c) Find bounds for $\frac{1}{1 + \sin^2\left(\frac{74}{x-3}\right)}$. Plot all three functions.
 - (d) Find bounds for $\frac{x-3}{1 + \sin^2\left(\frac{74}{x-3}\right)}$. Plot all three functions. Are your bounds actually bounds? Make sure they don’t cross over each other.
 - (e) What does this tell you about $\lim_{x \rightarrow 3} \frac{x}{1 + \sin^2\left(\frac{74}{x-3}\right)}$?
4. Use the same process for $(x + 1) \cos^2\left(\frac{32+x}{x+1}\right)$ (with domain $[-2, 0]$).
5. Use the same process for $(x + 1)^2 \cos^2\left(\frac{32+x}{x+1}\right)$. What’s different here?