Lab 4 Thursday February 22

Trigonometry

1. Define a function `h[x_]:=Sin[1/x]`. What is `h[0]`? Plug it into Mathematica and see what answer you get.
2. Plug in about ten numbers getting close to 0. What happens?
3. Plot a graph of `h[x]` with domain $[-1, 1]$. What happens near zero? Try using smaller domains like $[-.1, .1]$ or $[-.01, .01]$.
4. Use the `Limit` command to compute the limit at 0. What do you think this means?
5. Define `j[x_]:=x * Sin[1/x]`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
6. Plot a graph of `j[x]` with domain $[-1, 1]$. What happens near zero? Plot more graphs with smaller domains.
7. Use the `Limit` command to compute the limit at 0.
8. Define `k[x_]:=Sin[x]/x`. What do you think will happen near zero? Plug in some points near 0 to refine your guess.
9. Plot a graph of `k[x]` with domain $[-\pi, \pi]$. What happens near zero? Plot more graphs with smaller domains. Also plot some larger domains. Do these pictures match your expectations?
10. Use the `Limit` command to compute the limit at 0.

The Squeeze Theorem

A principle called the "Squeeze Theorem" or the "Two Policemen Theorem" allows us to compute the limit of a function we don't like by "trapping" or "squeezing" it between two functions which do. In the rest of this lab we'll visualize a few examples.

1. Earlier in this lab, we considered the function `x*Sin[1/x]` and its limit at zero. Now we want to generate better understanding of its behavior there.
   (a) Plot `x * Sin[1/x]` on the same graph as `Abs[x]` and `-Abs[x]`. (Bonus: color-code the graph so `x*Sin[1/x]` is one color and the other two graphs are a different color, using the PlotStyle command).
   (b) Shrink the domain around the point $x = 0$ to see what happens.
   (c) What are the limits of $|x|$ and $-|x|$ at 0? What does this tell us about the limit of $x \sin(1/x)$?
2. We also looked at the important limit `Sin[x]/x`.
   (a) As we did before, plot a graph of `Sin[x]/x`, along with the functions `1` and `Cos[x]`. (Again, for a bonus, color-code your graph).
   (b) Shrink the domain around $x = 0$ to see what happens. What are the limits of 1 and of $\cos(x)$ at 0?
3. Now we'll practice finding these bounds.
   (a) Using the fact that $-1 \leq \sin(a) \leq 1$, find upper and lower bounds for $\sin\left(\frac{74}{x-3}\right)$.
   (b) Use your answer to find bounds for $\sin^2\left(\frac{74}{x-3}\right)$ and then for $1 + \sin^2\left(\frac{74}{x-3}\right)$. Plot all three functions on the domain `{x,2,4}`.
   (c) Find bounds for $\frac{1}{1+\sin^2\left(\frac{74}{x-3}\right)}$. Plot all three functions.
   (d) Find bounds for $\frac{x-3}{1+\sin^2\left(\frac{74}{x-3}\right)}$. Plot all three functions. Are your bounds actually bounds? Make sure they don't cross over each other.
   (e) What does this tell you about $\lim_{x\to 3} \frac{x}{1+\sin^2\left(\frac{74}{x-3}\right)}$?
4. Use the same process for $(x + 1)\cos^2\left(\frac{32+x}{x+1}\right)$ (with domain $[-2, 0]$).
5. Use the same process for $(x + 1)^2 \cos^2\left(\frac{32+x}{x+1}\right)$. What's different here?