

**Problem 1.**

Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \rightarrow -2} \frac{x^2 + 6x + 8}{2(x+4)(x+2)} =$$

(b)

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9 - x}$$

(c)

$$\lim_{x \rightarrow -\infty} \frac{3x^3 + \sqrt[3]{x}}{\sqrt{9x^6 + 2x^2 + 1} + x}$$

(d)

$$\lim_{x \rightarrow 1^+} \frac{|x-1|}{x-1} =$$

**Problem 2.**

Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \rightarrow 1} \frac{\sin^2(x-1)}{(x-1)^2} =$$

(b)

$$\lim_{x \rightarrow -2} \frac{x^2 + 6x + 9}{2(x+4)(x+2)} =$$

(c)

$$\lim_{x \rightarrow \pi} \frac{\sin(x)}{x} =$$

(d)

$$\lim_{x \rightarrow 3} \frac{x-5}{(x-3)^2} =$$

**Problem 3.** (a) Using the Squeeze Theorem, show that

$$\lim_{x \rightarrow 3} \frac{x-3}{1 + \sin^2\left(\frac{2\pi + e + 7}{x-3}\right)} = 0.$$

(b) Let

$$g(x) = \begin{cases} \frac{x^2-1}{x-1} & x > 0 \\ x^2 + 1 & x < 0 \end{cases}$$

If possible, define an extension of  $g$  that is continuous at all real numbers.

**Problem 4.** (a) Show that the polynomial  $x^4 - 6x - 2$  has two real roots, that is, there are two (different!) real numbers  $a$  and  $b$  such that  $a^4 - 6a - 2 = b^4 - 6b - 2 = 0$ .

(b) Directly from the definition of derivative, compute  $f'(x)$  if  $f(x) = \sqrt{x+3}$ .

**Problem 5.** Compute the following derivatives using only the definition of derivative.

(a) Derivative of  $f(x) = x^2 + \sqrt{x}$  at  $x = 2$ .

(b) Derivative of  $g(x) = \frac{1}{x+1}$  at  $x = 1$ .