PRACTICE Comprehensive Exam **Student Number AXXXXXXX**: ____________________

Department of Mathematics Tuesday, March 15, 2016

Closed book. Closed notes. NO CALCULATORS. Time allowed: 3 hours for 5 sections (proportionally less if taking fewer than 5 sections). In other words, 36 minutes for each section taken. Please write very legibly and cross out all scratch work.

Calculus 1

1. ______ 2. ______ 3. ______ 4. ______ 5. ______ Total: ______

Calculus 2

6. ______ 7. ______ 8. ______ 9. ______ 10. ______ Total: ______

Multivariable Calculus

11. ______ 12. ______ 13. ______ 14. ______ 15. ______ Total: ______

Linear Algebra

16. ______ 17. ______ 18. ______ 19. ______ 20. ______ Total: ______

Discrete Mathematics

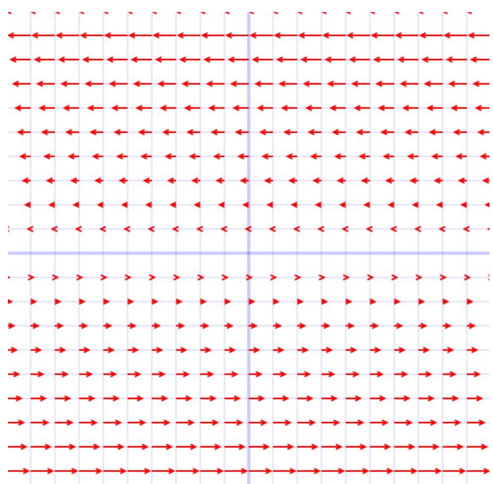
21. ______ 22. ______ 23. ______ 24. ______ 25. ______ Total: ______

Multivariable Calculus

11. Evaluate the line integral along the curve C where $C : y^2 = x^3$ from $(1, -1)$ to $(1, 1)$.

$$\int_C (y - x)\, dx + x^2 y\, dy$$

12. Consider the vector field $\vec{F}(x, y)$ below, estimate the signs of the following line integrals. EXPLAIN YOUR ANSWERS.



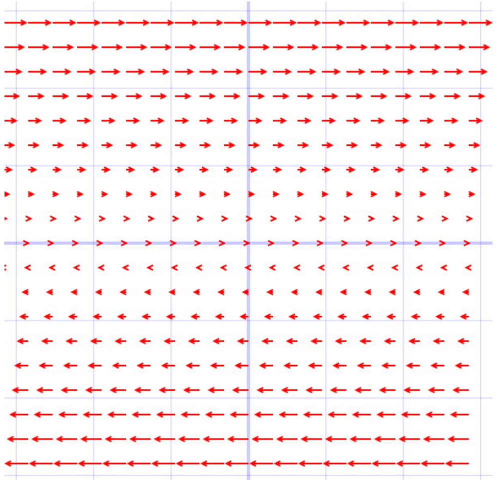
(a) $\int_{C_1} \vec{F} \cdot d\vec{x}$ where C_1 is the path along a circle of radius 1 centered at the origin, traversed in the counter-clockwise direction.

(b) $\int_{C_2} \vec{F} \cdot d\vec{x}$ where C_2 is the straight line path from $(1, 1)$ to $(-1, -1)$

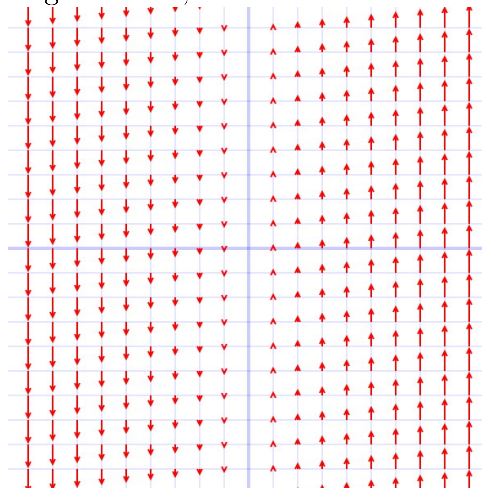
(c) $\int_{C_3} \vec{F} \cdot d\vec{x}$ where C_2 is the straight line path from $(1, -1)$ to $(-1, -1)$

(d) $\int_{C_4} \vec{F} \cdot d\vec{x}$ where C_2 is the straight line path from $(1, -1)$ to $(1, 1)$

13. Consider the two vector fields depicted in the figure below, labelled Field $\vec{A}$ and Field $\vec{B}$.



FIELD $\vec{A}$



FIELD $\vec{B}$

One of these two vector fields is a conservative field, i.e. can be written as the gradient of some function $\vec{\nabla} f$. Identify the conservative field, explain your selection and find the potential function $f(x, y)$ which produces the vector field you have selected.

14. Find the critical points of $f(x, y) = x^2 y + 2y^2 - 2xy + 6$ and use the second derivative test to classify them. (You do NOT need to identify global extrema of this function.)

15. Find three positive numbers whose sum is 27 and such that the sum of their squares is as small as possible.

16. Find the global maximum and global minimum of the function $f(x, y) = 4x + 2y$ on the region $x^2 + 4y^2 \leq 9$, if they exist.

17. Find a point on the surface $z = 3x^2 - y^2$ at which the tangent plane is parallel to the plane $6x + 4y - z = 5$.

18. Show that the surfaces

$$z = \sqrt{x^2 + y^2}$$

and

$$z = \frac{1}{10}(x^2 + y^2) + \frac{5}{2}$$

intersect at $(3, 4, 5)$ and have a common tangent plane at that point.

19. Let G be the solid in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the coordinate planes.

$$\mathcal{I} = \iiint_G xyz\, dV$$

(a) Write down a definite integral to evaluate $\mathcal{I}$ using two different coordinate systems. rectangular coordinates;

(b) Evaluate one of your integrals to obtain the exact value of $\mathcal{I}$.

20. Evaluate the integral by first reversing the order of integration.

$$\int_0^2 \int_{y/2}^1 \cos(x^2)\, dx\, dy$$