

Derivatives Practice Sheet Solutions

Compute the following derivatives, for practice. Do not simplify!

1.

$$\sqrt[5]{\frac{x^2 \sin(3x)}{\tan(x)}}$$

$$\frac{1}{5} \left(\frac{x^2 \sin(3x)}{\tan(x)} \right)^{-4/5} \frac{(2x \sin(3x) + x^2 \cos(3x)3) \tan(x) - \sec^2(x)x^2 \sin(3x)}{\tan^2(x)}$$

2.

$$\tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1).$$

$$\begin{aligned} \frac{d}{dx} \tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1) &= 4 \tan^3(\sqrt[3]{x^5 + x^3 + 2} + 1) \cdot \sec^2(\sqrt[3]{x^5 + x^3 + 2} + 1) \\ &\quad \cdot (\sqrt[3]{x^5 + x^3 + 2} + 1)' \\ &= 4 \tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1) \sec^2(\sqrt[3]{x^5 + x^3 + 2} + 1) \\ &\quad \cdot \left(\frac{1}{3} (x^5 + x^3 + 2)^{-2/3} \cdot (5x^4 + 3x^2) \right). \end{aligned}$$

3.

$$\cos \left(\frac{x^2 - \sqrt{5x^2 + 1}}{x^4 + \sin(x/2)} \right)$$

$$\begin{aligned} -\sin \left(\frac{x^2 - \sqrt{5x^2 + 1}}{x^4 + \sin(x/2)} \right) \cdot \\ \frac{(2x - \frac{1}{2}(5x^2 + 1)^{-1/2}10x)(x^4 + \sin(x/2)) - (4x^3 + \cos(x/2)\frac{1}{2})(x^2 - \sqrt{5x^2 + 1})}{(x^4 + \sin(x/2))^2} \end{aligned}$$

4.

$$\left(\sin(4\sqrt{5 \tan(\sqrt[3]{\csc(3x^2)})}) \right)^5$$

$$\begin{aligned} 5 \left(\sin(4\sqrt{5 \tan(\sqrt[3]{\csc(3x^2)})}) \right)^4 \cdot \cos((4\sqrt{5 \tan(\sqrt[3]{\csc(3x^2)})})) \cdot \\ 4 \cdot \frac{1}{2} (5 \tan(\sqrt[3]{\csc(3x^2)}))^{-1/2} \cdot 5 \sec^2(\sqrt[3]{\csc(3x^2)}) \cdot \\ \frac{1}{3} (\csc(3x^2))^{-2/3} \cdot (-1) \csc(3x^2) \cot(3x^2) \cdot 6x \end{aligned}$$

5.

$$\frac{\tan(x) + \sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}}}{x^4 \sin(x/5) \cos(x)}$$

$$\frac{\left(\sec^2(x) + \frac{1}{3} \left(\frac{7\sec(x)+x}{\sqrt{2\sin(x)}} \right)^{-2/3} \cdot \frac{(7\sec(x)\tan(x)+1)\sqrt{2\sin(x)} - \frac{1}{2}(2\sin(x))^{-1/2} 2\cos(x)(7\sec(x)+x)}{2\sin(x)} \right) (x^4 \sin(x/5) \cos(x))}{x^8 \sin^2(x/5) \cos^2(x)}$$

$$\frac{(4x^3 \sin(x/5) \cos(x) + x^4 \cos(x/5)(1/5) \cos(x) - x^4 \sin(x/5) \sin(x)) \left(\tan(x) + \sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}} \right)}{x^8 \sin^2(x/5) \cos^2(x)}$$

6.

$$\cot^{4/3} \left(\frac{\sqrt{\sin(3x/2) + 1} + x}{\sin(7x^{5/4})} \right)$$

$$\frac{\frac{4}{3} \cot^{1/3} \left(\frac{\sqrt{\sin(3x/2) + 1} + x}{\sin(7x^{5/4})} \right) \cdot (-1) \csc^2 \left(\frac{\sqrt{\sin(3x/2) + 1} + x}{\sin(7x^{5/4})} \right)}{\left(\frac{1}{2}(\sin(3x/2) + 1)^{-1/2} \cos(3x/2)3/2 + 1 \right) \sin(7x^{5/4}) - \cos(7x^{5/4})(7 \cdot \frac{5}{4}x^{1/4}) \left(\sqrt{\sin(3x/2) + 1} + x \right)}$$

$$\frac{\sin^2(7x^{5/4})}{\sin^2(7x^{5/4})}$$

7.

$$\sqrt[7]{\csc \left(\frac{\cos((x^3 + 1)^2 \sin(x)) + \tan(x)}{\sqrt{5x + 3\sin(x)}} \right)}$$

$$\frac{1}{7} \left(\csc \left(\frac{\cos((x^3 + 1)^2 \sin(x)) + \tan(x)}{\sqrt{5x + 3\sin(x)}} \right) \right)^{-6/7}$$

$$\cdot (-1) \csc^2 \left(\frac{\cos((x^3 + 1)^2 \sin(x)) + \tan(x)}{\sqrt{5x + 3\sin(x)}} \right) \cdot \cot \left(\frac{\cos((x^3 + 1)^2 \sin(x)) + \tan(x)}{\sqrt{5x + 3\sin(x)}} \right)$$

$$\frac{(-\sin((x^3 + 1)^2 \sin(x))(2(x^3 + 1)3x^2 \sin(x) + (x^3 + 1)^2 \cos(x)) + \sec^2(x)) \sqrt{5x + 3\sin(x)}}{5x + 3\sin(x)}$$

$$\frac{\frac{1}{2}(5x + 3\sin(x))^{-1/2}(5 + 3\cos(x))(\cos((x^3 + 1)^2 \sin(x)) + \tan(x))}{5x + 3\sin(x)}$$