Math 114 Fall 2019
Calculus I HW 2 Solutions
Due Wednesday, September 11

1. Stewart 1.5.16
2. Let

$$f(x) = \begin{cases} x+3 & x > 2 \\ x^2+1 & x < 2 \end{cases}$$

Define a function that extends f and is continuous at all real numbers.

Solution: Define

$$f_F(x) = \begin{cases} x+3 & x > 2 \\ x^2+1 & x < 2 \\ 5 & x = 2 \end{cases}$$

Then f_F is continuous at 2 since $\lim_{x\to 2^-} f_F(x) = \lim_{x\to 2} x^2+1 = 5$ and $\lim_{x\to 2^+} f_F(x) = \lim_{x\to 2} x+3 = 5$.

3. Let

$$g(x) = \begin{cases} x^2-5 & x > -1 \\ 4x & x < -1 \end{cases}$$

Define a function that extends g and is continuous at all real numbers.

Solution: Define

$$g_F(x) = \begin{cases} x^2-5 & x > -1 \\ 4x & x < -1 \\ -4 & x = -1 \end{cases}$$

Then g_F is continuous at -1 since $\lim_{x\to -1^-} g_F(x) = \lim_{x\to -1} 4x = -4$ and $\lim_{x\to -1^+} g_F(x) = \lim_{x\to -1} x^2-5 = -4$.

4. Stewart 1.5.30
5. Stewart 1.4.34
6. Stewart 1.4.36

7. ($\star$) Using the squeeze theorem, show that

$$\lim_{x\to-2} \frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)} = 0.$$

Solution: We observe that

$$\begin{aligned} -1 \le \sin\left(\frac{1}{x+2}\right) &\le 1 \\ 1 \le 2+\sin\left(\frac{1}{x+2}\right) &\le 3 \\ 1 \ge \frac{1}{2+\sin\left(\frac{1}{x+2}\right)} &\ge \frac{1}{3} \ge -1 \\ |x+2| \ge \frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)} &\ge -|x+2| \end{aligned}$$

(Note: using $|x+2|/3$ does not work here at all; it doesn't bound things below on the left.)

Then we compute $\lim_{x\to-2}|x+2| = 0$ and $\lim_{x\to-2} -|x+2| = 0$, so by the Squeeze Theorem,

$$\lim_{x\to-2} \frac{x+2}{2+\sin\left(\frac{1}{x+2}\right)} = 0.$$

8. Stewart 1.4.50

9. Stewart 1.4.52 (Hint: what trig identities do we know? Can we make one of them show up?)

10. Stewart 1.4.54