Math 114 Fall 2019
Calculus I HW 3 Solutions
Due Wednesday, September 18

1. Compute $\lim_{x\to 1} \frac{x^2+3}{x-1}$.

   Solution: The limit of the top is 4 and the limit of the bottom is zero, so the limit is $\pm\infty$.

   (Since the bottom can be both positive and negative, we can't say more than this.)

2. Compute $\lim_{x\to -3} \frac{x-4}{(x+3)^2}$.

   Solution: The limit of the top is -7 and the limit of the bottom is zero, so the limit is $\pm\infty$.

   Moreover, the bottom is always positive and the top is negative, so the limit is in fact $-\infty$.

3. Stewart 1.6.16

4. Stewart 1.6.18

5. Stewart 1.6.20

6. Stewart 1.6.22

7. Stewart 1.6.24

8. ($\star$) Stewart 1.6.28

9. Compute $\lim_{x\to -\infty} \frac{x^3+1}{\sqrt{x^6+x^4+1}}$.

   Solution:

$$\begin{aligned}\lim_{x\to-\infty} \frac{x^3+1}{\sqrt{x^6+x^4+1}} &= \lim_{x\to-\infty} \frac{1+1/x^3}{\sqrt{x^6+x^4+1}/(-\sqrt{x^6})} \\ &= \lim_{x\to-\infty} \frac{1+1/x^3}{-\sqrt{1+1/x^2+1/x^6}} \\ &= \frac{1+0}{-\sqrt{1+0+0}} = -1.\end{aligned}$$

10. Compute $\lim_{x\to+\infty} x^2 - x$.

Solution:

$$\begin{aligned}\lim_{x\to+\infty} x^2 - x &= \lim_{x\to+\infty} \frac{x^2-x}{1} \\ &= \lim_{x\to+\infty} \frac{1-1/x}{1/x^2}.\end{aligned}$$

The limit of the top is 1 and the limit of the bottom is 0, so the limit is $\pm\infty$. Further, the bottom is always positive, so the limit is in fact $+\infty$.

11. Compute $\lim_{x\to+\infty} 2x - \sqrt{4x^2+3x+1}$

Solution:

$$\begin{aligned}\lim_{x\to+\infty} 2x - \sqrt{4x^2+3x+1} &= \lim_{x\to+\infty} \frac{4x^2-(4x^2+3x+1)}{2x+\sqrt{4x^2+3x+1}} \\ &= \lim_{x\to+\infty} \frac{-3x-1}{2x+\sqrt{4x^2+3x+1}} \\ &= \lim_{x\to+\infty} \frac{-3-1/x}{2+\sqrt{4+3/x+1/x^2}} \\ &= \frac{-3-0}{2+\sqrt{4+0+0}} = \frac{-3}{4}.\end{aligned}$$

12. If $f(x) = \sqrt[3]{x}$, then it is a fact that $f'(27) = \frac{1}{27}$. Use linear approximation to estimate $\sqrt[3]{25}$ and $\sqrt[3]{30}$.

Solution:

$$\begin{aligned}f(25) &\approx f(27) + \frac{1}{27}(25-27) = 3 + \frac{1}{27}(-2) = \frac{79}{27} \approx 2.925 \\ f(30) &\approx f(27) + \frac{1}{27}(30-27) = 3 + \frac{1}{27}(3) = \frac{28}{9} \approx 3.\overline{11}.\end{aligned}$$

13. If $g(x) = x^2$, then it is a fact that $g'(3) = 6$. Use linear approximation to estimate 2.8^2 and 3.2^2.

(Also: you don't need to submit this answer, but think about how your approximations relate to $g(3)$, and why any symmetries you notice are there.)

Solution:

$$\begin{aligned}g(2.8) &\approx g(3) + 6(2.8-3) = 9 + 6\cdot(-.2) = 7.8. \\ g(3.2) &\approx g(3) + 6(3.2-3) = 9 + 6\cdot .2 = 10.2.\end{aligned}$$

The two approximations are symmetric around $g(3) = 9$ because they are linear; increasing the input by .2 increases the output by 1.2, so decreasing the input by .2 should decrease the output by 1.2.