Math 114 Fall 2019
Calculus I HW 5 Solutions
Due Wednesday, March 6

For this homework you may compute derivatives using any tools we have developed in class.

1. Stewart 2.1.10
2. Stewart 2.1.12
3. Stewart 2.1.46
4. Stewart 2.1.48
5. Stewart 2.3.52
6. Stewart 2.3.56
7. Stewart 2.4.50
8. Stewart 2.4.54
9. Stewart 2.5.64
10. Stewart 2.5.66

11. Verify that $f(x) = \sin(x^2)$ is a solution to the differential equation $y'' = y'/x - 4x^2y$.

    Solution: We compute

    $$\begin{aligned} f'(x) &= 2x\cos(x^2) \\ f''(x) &= 2\cos(x^2) - 4x^2\sin(x^2) \end{aligned}$$

    so

    $$f''(x) = (2x\cos(x^2))/x - 4x^2(\sin(x^2)) = f'(x)/x - 4x^2 f(x).$$

12. Suppose $f(x) = ax^2 + bx + c$ satisfies $f(2) = 1, f'(2) = 2, f''(2) = 3$. Find $f(x)$.

    Solution: We have $1 = f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$; we have $2 = f'(2) = 2a(2) + b = 4a + b$; and we have $3 = f''(2) = 2a$. This tells us that $a = 3/2$. Thus $2 = 6+b$ so $b = -4$. Finally, we get $1 = 6-8+c$ so $c = 3$. Thus $f(x) = 3x^2/2-4x+3$.

13. Prove that $f(x) = \sin(x) + x^2$ satisfies $f''(x) + f(x) = x^2 + 2$.

    Solution: We see that $f'(x) = \cos(x) + 2x$ and $f''(x) = -\sin(x) + 2$. Thus

    $$f''(x) + f(x) = -\sin(x) + 2 + \sin(x) + x^2 = x^2 + 2$$

    as desired.

14. Suppose the rate at which the concentration of a drug in the bloodstream decreases is proportional to the current concentration. Write a differential equation to express this model, and write a sentence or two explaining what each variable means and how the equation models this scenario.

    Solution:

    Let $C(t)$ be the concentration of a drug in the bloodstream at the time t; the units of $C(t)$ should be something like milligrams/liter, and the units of t are probably hours.

    Then we get the differential equation

    $$-C'(t) = kC(t).$$

    The negative sign isn't strictly necessary; if k is positive then we need the minus sign, and if k is negative we do not. But the minus sign is probably a better choice because it makes it easier to see what is happening.

15. Suppose the rate at which an epidemic spreads through a community is jointly proportional to the number of people who have caught the disease and to the number of people who have not. Write a differential equation to express this model, and write a sentence or two explaining what each variable means and how the equation models this scenario.

    Solution: Let $D(t)$ be the number of people who have caught the disease, with t having units of days and $D(t)$ having units of people. Let M be the total number of people. Then we get the equation

    $$D'(t) = kD(t)(M - D(t)).$$

16. Suppose the rate at which people hear about a piece of news is proportional to the number of people who have *not* heard about it. Write a differential equation to express this model, and write a sentence or two explaining what each variable means and how the equation models this scenario.

    Solution: Let $P(t)$ be the number of people who have heard the news at a time t. Then the input units are something like "days" and the output units are "people". Let M be the total number of people (again, the units are "people").

    Then the number of people who have not heard the news at time t is $M - P(t)$, and so we get the differential equation

    $$P'(t) = k(M - P(t).$$

    Alternatively, we could let $Q(t)$ be the number of people who have not heard the news; then we have

    $$-Q'(t) = kQ(t).$$