Math 114 Fall 2019
Calculus I HW 6 Solutions
Due **Friday**, October 18

For this homework you may compute derivatives using any tools we have developed in class.

1. Stewart 3.4.6 (ab)

2. Stewart 3.4.8

3. Stewart 3.4.14

4. Stewart 3.4.18(a)

5. Suppose we know that $f''(t) = -f(t)$, and we know that $f(0) = -1$ and $f(\pi/3) = 4$. Find a formula for $f(t)$.

   Solution: We know that $f(t) = a\sin(t) + b\cos(t)$. Then we have

$$\begin{aligned} -1 &= f(0) = a\sin(0) + b\cos(0) = b \\ 4 &= f(\pi 3) = a\sin(\pi/3) + b\cos(\pi/3) \\ &= a\sqrt{3}/2 + b/2 = a\sqrt{3}/2 - b/2 \\ 8 &= a\sqrt{3} - 1 \\ a &= 9/\sqrt{3} = 3\sqrt{3}. \end{aligned}$$

   Thus
$$f(t) = 3\sqrt{3}\sin(t) - \cos(t).$$

6. Suppose $f'(x) = \frac{1}{2}f(x)$ and $f(1) = 3$. Use Euler's method to approximate $f(4)$ using three steps.

   Solution: We have

$$\begin{aligned} f(2) &\approx f'(1)(2-1) + f(1) = 3/2 \cdot 1 + 3 = 4.5 \\ f(3) &\approx f'(2)(3-2) + f(2) \approx 9/4 + 9/2 = 27/4 \\ f(4) &\approx f'(3)(4-3) + f(3) \approx 27/8 + 27/4 = 81/8. \end{aligned}$$

7. Suppose $f'(x) = 4 - \frac{f(x)}{x}$, and $f(2) = 2$. Use Euler's method and four steps to approximate $f(4)$.

   Solution:

$$\begin{aligned}
f(5/2) &\approx f'(2)(.5) + f(2) = 3/2 + 2 = 7/2 \\
f(3) &\approx f'(5/2)(1/2) + f(5/2) \\
&\approx \left(4 - \frac{7/2}{5/2}\right)/2 + 7/2 = 13/10 + 7/2 = 24/5 \\
f(7/2) &\approx f'(24/5)(1/2) + f(3) \\
&\approx \left(4 - \frac{24/5}{3}\right)/2 + 24/5 = 6/5 + 24/5 = 6 \\
f(4) &\approx f'(7/2)(1/2) + f(7/2) \\
&\approx \left(4 - \frac{6}{7/2}\right)/2 + 6 = 8/7 + 6 = 50/7.
\end{aligned}$$

8. Suppose $f'(x) = xf(x)$, and $f(0) = 3$. Use four steps to estimate $f(4)$.

   Solution:

$$\begin{aligned}
f(1) &\approx f'(0)(1-0) + f(0) = 0 \cdot 1 + 3 = 3 \\
f(2) &\approx f'(1)(2-1) + f(1) \approx (1 \cdot 3) \cdot 1 + 3 = 6 \\
f(3) &\approx f'(2)(3-2) + f(2) \approx (2 \cdot 6) \cdot 1 + 6 = 18 \\
f(4) &\approx f'(3)(4-3) + f(3) \approx (3 \cdot 18) \cdot 1 + 18 = 72.
\end{aligned}$$

9. Suppose we have the differential equation $y'' = 3y$, and $y(0) = 2, y'(0) = -1$. Use three steps of Euler's method to estimate $y(3)$.

   Solution:

$$\begin{aligned}
y(1) &\approx y(0) + y'(0)(1-0) = 2 - 1 = 1 \\
y'(1) &\approx y'(0) + y''(0)(1-0) = -1 + 6 = 5 \\
y(2) &\approx y(1) + y'(1)(2-1) \approx 1 + 5 = 6 \\
y'(2) &\approx y'(1) + y''(1)(2-1) \approx 5 + 3 = 8 \\
y(3) &\approx y(2) + y'(2)(3-2) = 6 + 8 = 14.
\end{aligned}$$

10. Stewart 3.3.28

11. Stewart 3.3.32

12. Stewart 3.3.46

13. Stewart 2.6.10

14. Stewart 2.6.20

15. Stewart 2.6.22

16. Suppose we know that $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$, and we know that $y(0) = 1/2$. Estimate $y(.1)$ using a linear approximation.

Solution: Implicit differentiation gives

$$\begin{aligned} 2x + 2yy' &= 2(2x^2 + 2y^2 - x)(4x + 4yy' - 1) \\ y' &= 2(0 + 1/2 - 0)(0 + 2y' - 1) \\ y' &= 2y' - 1 \\ y' &= 1 \end{aligned}$$

so we get the linear approximation

$$\begin{aligned} y &\approx 1/2 + 1(x - 0) \\ y(.1) &\approx 1/2 + 1(.1) = .6. \end{aligned}$$