Lab 3 Tuesday September 10

The Squeeze Theorem

A principle called the "Squeeze Theorem" or the "Two Policemen Theorem" allows us to compute the limit of a function we don't like by "trapping" or "squeezing" it between two functions which do. In the rest of this lab we'll visualize a few examples. We'll discuss more in class soon.

1. In class we have been studying the function `x*Sin[1/x]` and its limit at zero. Now we want to generate better graphical understanding of its behavior there.

   (a) Plot the function $x\sin(1/x)$ with the code `Plot[x * Sin[1/x],{x,-1,1}]`.

   (b) Plot the function along with x and $-x$ with the code `Plot[{x * Sin[1/x],x,-x},{x,-1,1}]`.

   (c) Now plot $x\sin(1/x)$ on the same graph as $|x|$ and $-|x|$, with the code `Plot[{x * Sin[1/x], Abs[x],-Abs[x]},{x,-1,1}]`

   (d) Plot the function again with smaller domains centered at $x = 0$ to see what happens.

   (e) How does this relate to our squeeze theorem arguments from class?

2. (a) Plot the function `x(8 Sin[1/x]-5)`. What is the limit as x approaches 0?

   (b) We can see that $-13 \leq 8\sin(1/x)-5 \leq 3$. What bounds does this give us for $x\,(8\sin(1/x)-5)$? Plot those bounds together with the original function. Do they work?

   (c) Now plot `Abs[x(8 Sin[1/x]-5)]`, together with the absolute values of your bounds. What do you see? What would make a good lower bound here? What's the smallest upper bound that works, and why?

   (d) What would change if we looked at `x(8 Sin[1/x]+5)` instead?

3. (a) Using the fact that $-1 \leq \cos(a) \leq 1$ for any a, find upper and lower bounds for $\cos\left(\frac{32+x}{x+1}\right)$. Find a number that's always bigger than this function, and another that's always smaller.

   (b) Use your answer to find bounds for $\cos^2\left(\frac{32+x}{x+1}\right)$. Again, you should find a number that's always bigger and one that's always smaller.

   Plot this function to check your answer, with `Plot[Cos[(32+x)/(x+1)]^2,{x,-2,0}]`. Notice you need to use `Cos[(32+x)/(x+1)]^2` and *not* `Cos^2[(32+x)/(x+1)]`

   (c) Now find bounds for $(x+1)\cos^2\left(\frac{32+x}{x+1}\right)$. You should have a *function* that's always bigger and a function that's always smaller.

   (d) Plot all three functions from the previous part with domain $[-2, 0]$. Is your upper bound actually always above the function? Is your lower bound always below? Make sure the bounds don't cross *through* the function. If they do, can you fix this?

   (e) What does this picture suggest about $\lim x \to -1(x+1)\cos^2\left(\frac{32+x}{x+1}\right)$?

4. Do all the same steps for a new function: $(x+1)^2\cos^2\left(\frac{32+x}{x+1}\right)$. What's very different about the graph of this function?

Asymptotes

Look at the following functions, and determine:

1. Where do you expect to find vertical asymptotes? What kinds?
2. What happens when the inputs get large? Do you expect horizontal asymptotes?
3. Do you expect to find any roots?
4. Plot the functions with the Mathematica `Plot` command. Remember to include a domain!

Coding tip: The horizontal asymptotes might be easier to see if the domain is large.

Coding Tip: You can download the "Plot Piecewise Code" from the Moodle site to get a much better view of these graphs, using `PlotPiecewise` instead of `Plot`

Coding tip: Remember you can use the `PlotRange` option with `Plot[f[x],{x,-5,5},PlotRange->{-15,15}]` (or with different numbers) to fix the height shown on the graph. This can be useful if too much information is hidden by the scale.

(a) `1/(x^2-5x+6)`

(b) `1/(x^4+9x^3+29x^2+39x+18)`

(c) `(x-1)^(-2) (x-2)^(-2)`

(d) `(x-1)^(2) / (x-2)^2`

(e) `(x+1)/(Abs[x]-1)`

(f) `(x+1)/Abs[x -1]`
(Why do these two look so different?)

(g) `1/(Abs[x] * x + 1)`

(h) `x/Sqrt[x^2+1]`

(i) `(x^(3/2) + x)/(Sqrt[x^3+x+5])`

(j) `x/Sqrt[x^3+x+5]`

(k) `Sqrt[x^7+x]/(x^3+1)`

(l) `Tan[x]`

(m) `x * Tan[x]`

(n) `Csc[x]`

(o) `x * Csc[x]`

(p) `x -x ^2`

(q) `1/(x - x^2)`

(r) `Sqrt[x^2+1]-x`