Math 114 Fall 2019
Calculus I Practice Homework 4.5 Solutions
Do not turn in

Please be careful about parentheses, especially in the chain rule problems. Leaving out parentheses can make your answers actively wrong, and we will take off points for bad parenthesization.

Note that the odd-numbered problems in Stewart have solutions in the back of the book.

I definitely don't expect you to finish all of these, especially the last couple. But if you want practice, there's plenty here.

1. Naming each derivative rule used explicitly, compute $\frac{d}{dx}\left(x^2\tan(x)+x\right)$.

   Solution:

$$\begin{aligned}
\frac{d}{dx}\left(x^2\tan(x)+x\right) &= \frac{d}{dx}x^2\tan(x)+\frac{d}{dx}x && \text{Additivity}\\
&= (x^2)'\tan(x)+x^2\tan'(x)+x' && \text{Product rule}\\
&= 2x\tan(x)+x^2\tan'(x)+1 && \text{Power rule}\\
&= 2x\tan(x)+x^2\sec^2(x)+1 && \text{Trig rules}
\end{aligned}$$

2. Naming each derivative rule used explicitly, compute $\frac{d}{dx}\sqrt{x}\cos(x)\csc(x)$.

   Solution:

$$\begin{aligned}
\frac{d}{dx}\sqrt{x}\cos(x)\csc(x) &= (\sqrt{x})'\cos(x)\csc(x)+\sqrt{x}(\cos(x)\csc(x))'\\
&\qquad \text{Product rule}\\
&= (\sqrt{x})'\cos(x)\csc(x)+\sqrt{x}(\cos'(x)\csc(x)+\csc'(x)\cos(x))\\
&\qquad \text{Product rule}\\
&= \frac{1}{2\sqrt{x}}\cos(x)\csc(x)+\sqrt{x}(\cos'(x)\csc(x)+\csc'(x)\cos(x))\\
&\qquad \text{Power rule}\\
&= \frac{1}{2\sqrt{x}}\cos(x)\csc(x)+\sqrt{x}(-\sin(x)\csc(x)-\csc(x)\cot(x)\cos(x))\\
&\qquad \text{Trig rules}
\end{aligned}$$

3. Naming each derivative rule used explicitly, compute $\frac{d}{dx}\frac{\sqrt{x^2+1}}{\sin(x)}$

   Solution:

$$\begin{aligned}
\frac{d}{dx}\frac{\sqrt{x^2+1}}{\sin(x)} &= \frac{\frac{d}{dx}\sqrt{x^2+1}\cdot\sin(x)-\sin'(x)\sqrt{x^2+1}}{\sin^2(x)} && \text{Quotient rule}\\
&= \frac{\frac{d}{dx}\sqrt{x^2+1}\cdot\sin(x)-\cos(x)\sqrt{x^2+1}}{\sin^2(x)} && \text{Trig rules}\\
&= \frac{\frac{1}{2}(x^2+1)^{-1/2}\cdot(x^2+1)'\sin(x)-\cos(x)\sqrt{x^2+1}}{\sin^2(x)} && \text{Chain rule and power rule}\\
&= \frac{\frac{\sin(x)}{2\sqrt{x^2+1}}((x^2)'+1')-\cos(x)\sqrt{x^2+1}}{\sin^2(x)} && \text{Additivity}\\
&= \frac{\frac{\sin(x)}{2\sqrt{x^2+1}}(2x+0)-\cos(x)\sqrt{x^2+1}}{\sin^2(x)} && \text{Power rule and constants}
\end{aligned}$$

4. Stewart 2.4.11
5. Stewart 2.4.26
6. Stewart 2.4.27
7. Stewart 2.5.1
8. Stewart 2.5.3
9. Stewart 2.5.5
10. Stewart 2.5.7
11. Stewart 2.5.13
12. Stewart 2.5.15
13. Stewart 2.5.21
14. Stewart 2.5.23
15. Stewart 2.5.41
16. Stewart 2.8.11
17. Stewart 2.8.13
18. $\frac{d}{dx}\sqrt[5]{\frac{x^2\sin(3x)}{\tan(x)}} =$

    Solution:

$$\frac{1}{5}\left(\frac{x^2\sin(3x)}{\tan(x)}\right)^{-4/5}\frac{(2x\sin(3x)+x^2\cos(3x)3)\tan(x)-\sec^2(x)x^2\sin(3x)}{\tan^2(x)}$$

19. $\dfrac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1). =$

Solution:

$$\begin{aligned}\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1) &= 4\tan^3(\sqrt[3]{x^5+x^3+2}+1)\cdot\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot(\sqrt[3]{x^5+x^3+2}+1)'\\ &= 4\tan^4(\sqrt[3]{x^5+x^3+2}+1)\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot\left(\frac{1}{3}(x^5+x^3+1)^{-2/3}\cdot(5x^4+3x^2)\right).\end{aligned}$$

20. $\dfrac{d}{dx}\cos\left(\dfrac{x^2-\sqrt{5x^2+1}}{x^4+\sin(x/2)}\right) =$

Solution:

$$-\sin\left((\frac{x^2-\sqrt{5x^2+1}}{x^4+\sin(x/2)}\right)\cdot \frac{(2x-\frac{1}{2}(5x^2+1)^{-1/2}10x)(x^4+\sin(x/2))-(4x^3+\cos(x/2)\frac{1}{2})(x^2-\sqrt{5x^2+1})}{(x^4+\sin(x/2))^2}$$

21. $\dfrac{d}{dx}\left(\sin(4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})})\right)^5 =$

Solution:

$$\begin{aligned}&5\left(\sin(4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})})\right)^4\cdot\cos((4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})}))\cdot\\ &\quad 4\cdot\frac{1}{2}(5\tan(\sqrt[3]{\csc(3x^2)}))^{-1/2}\cdot 5\sec^2(\sqrt[3]{\csc(3x^2)})\cdot\\ &\quad\frac{1}{3}(\csc(3x^2))^{-2/3}\cdot(-1)\csc(3x^2)\cot(3x^2)\cdot 6x\end{aligned}$$

22. $\dfrac{d}{dx}\dfrac{\tan(x)+\sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}}}{x^4\sin(x/5)\cos(x)} =$

Solution:

$$\frac{\left(\sec^2(x)+\frac{1}{3}\left(\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}\right)^{-2/3}\cdot\frac{(7\sec(x)\tan(x)+1)\sqrt{2\sin(x)}-\frac{1}{2}(2\sin(x))^{-1/2}2\cos(x)(7\sec(x)+x)}{2\sin(x)}\right)(x^4\sin(x/5)\cos(x)$$

$$-\frac{(4x^3\sin(x/5)\cos(x)+x^4\cos(x/5)(1/5)\cos(x)-x^4\sin(x/5)\sin(x))\left(\tan(x)+\sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}}\right)}{x^8\sin^2(x/5)\cos^2(x)}$$

23. $\dfrac{d}{dx}\cot^{4/3}\left(\dfrac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right) =$

Solution:

$$\frac{4}{3}\cot^{1/3}\left(\frac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right)\cdot(-1)\csc^2\left(\frac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right)$$
$$\cdot\frac{\left(\frac{1}{2}(\sin(3x/2)+1)^{-1/2}\cos(3x/2)3/2+1\right)\sin(7x^{5/4})-\cos(7x^{5/4})(7\cdot\frac{5}{4}x^{1/4})\left(\sqrt{\sin(3x/2)+1}+x\right)}{\sin^2(7x^{5/4})}$$

24. $\dfrac{d}{dx}\sqrt[7]{\csc\left(\dfrac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)} =$

Solution:

$$\frac{1}{7}\left(\csc\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)\right)^{-6/7}$$
$$\cdot(-1)\csc^2\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)\cdot\cot\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)$$
$$\cdot\frac{(-\sin((x^3+1)^2\sin(x))(2(x^3+1)3x^2\sin(x)+(x^3+1)^2\cos(x))+\sec^2(x))\sqrt{5x+3\sin(x)}}{5x+3\sin(x)}$$
$$-\frac{\frac{1}{2}(5x+3\sin(x))^{-1/2}(5+3\cos(x))(\cos\left((x^3+1)^2\sin(x)\right)+\tan(x))}{5x+3\sin(x)}$$