Problem 1. (a) Compute $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$

(b) Compute $\lim_{x \to 0} \frac{\sin(x^2)}{\sin^2(x)}$

(c) Compute $\lim_{x \to -\infty} \frac{3x}{\sqrt{4x^2 + 3}}$

(d) Compute $\lim_{x \to 0} \frac{e^x - \tan(x) - 1}{x^2}$

Problem 2. (a) Let

$$f(x) = \begin{cases} e^{x^2-1} & x > 1 \\ x^3 - 2x + 2 & x < 1 \end{cases}$$

If possible, define an extension of f that is continuous at all real numbers.

(b) Use the Squeeze Theorem to show that $\lim_{x \to 5} (x-5) \sin\left(\frac{x^2+1}{x-5}\right) = 0$.

(c) Suppose that if a car travels at v miles per hour then its fuel efficiency is $F(v) = 8 + 1.3v - .015v^2$ miles per gallon.

(i) What does the derivative $F'(v)$ represent, and what are its units?

(ii) Compute $F'(60)$. What does this tell you?

Problem 3. (a) **Directly from the definition**, compute $f'(1)$ where $f(x) = \sqrt{x+3}$.

(b) Compute $g'(x)$ where $g(x) = \ln\left|\dfrac{e^{\arctan(x^2)} - 5}{\sqrt[4]{x^2+1}}\right|$.

(c) Find a tangent line to the function $f(x) = \frac{e^x}{x}$ at the point given by $x = 2$.

Problem 4. (a) Let $g(x) = \sqrt[5]{x^9 + x^7 + x + 1}$. Find $(g^{-1})'(1)$.

(b) Write a tangent line to the curve $y^2 = x^{x\cos(x)}$ at the point $(\pi/2, -1)$.

(c) Find y' if $e^y + \ln(y) = x^2 + 1$.

Problem 5. (a) A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. Suppose we have an inverted conical water tank, of height 4m and radius 6m. Water is leaking out of a small hole at the bottom of the tank. If the current water level is 2m and the water level is dropping at $\frac{1}{9\pi}$ meters per minute, what volume of water leaks out every minute?

(b) Use two iterations of Newton's method, starting at 4, to estimate $\sqrt{15}$.

(c) Find all the critical points of $g(x) = \ln(x^3 + 9x^2 + 27x)$.

Problem 6. (a) If $f(x) = \sqrt{x} + \tan(\pi x)$, use a linear approximation centered at 4 to estimate $f(4.1)$.

(b) If $g(x) = \cos(x)$, use a quadratic approximation centered at 0 to estimate $g(.1)$.

(c) Let $g'(x) = g(x) + 3x$, and $g(2) = 4$. Use two steps of Euler's method to estimate $g(4)$. Is this an overestimate or an underestimate?

Problem 7.

(a) Determine whether $f(x) = x^2 + 2x + e^x$ is a solution to the differential equation $y'' - y' + 2x = 0$.

(b) A population of bacteria intially contains 90,000 bacteria, and after two weeks it will contain 180,000 bacteria. How long will it take the population to grow from its initial population to reach 150,000 bacteria?

(c) Find an antiderivative for $h(x) = \frac{1}{1+x} + \frac{1}{(1+x)^2}$.

Problem 8. (a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Ten miles from home you remember that you left the water running, which is costing you 90 cents an hour. Driving home at speed s miles per hour costs you $4(s/10)$ cents per mile. At what speed should you drive to minimize the total cost of gas and water?

(c) Classify the relative extrema of $h(x) = \sqrt[3]{x}(x + 4)$

Problem 9. (a) Find all the critical points of $g(x) = \dfrac{x^2 - 8}{x + 3}$

(b) If $-1 \leq f'(x) \leq 3$ and $f(0) = 0$, what can you say about $f(4)$? Assume f is continuous and differentiable.

(c) Prove that $x^2 - (e^2 + 1)\ln(x)$ has exactly two real roots.

Problem 10. (a) Let $F(0) = 2$ and $F'(x) = x^3 + x$. Use a modified Euler's method with three steps to estimate $F(3)$.

(b) Give a formula for the quadratic approximation of $g(x) = e^{x^2-1} + x$ near the point $a = 1$.

(c) Suppose you want to design a closed box with a square base and a volume of 2250in^3. The material for the top and bottom costs \$2 per square inch, and the material for the sides costs \$3 per square inch. What are the dimensions of the box with minimum possible cost?

Problem 11. Let $j(x) = x^4 - 14x^2 + 24x + 6$. We can compute $j'(x) = 4(x+3)(x-1)(x-2)$ and $j''(x) = 4(3x^2 - 7)$. Sketch a graph of j.

Your answer should discuss the domain, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

Problem 12. Let $g(x) = \arctan(x^2 + x)$. We can compute that $g'(x) = \frac{2x+1}{1+(x^2+x)^2}$ and

$$g''(x) = \frac{-6x^4 - 12x^3 - 8x^2 - 2x + 2}{(1 + (x^2 + x)^2)^2}.$$

Sketch a graph of g.

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.

(Note: concavity is hard on this problem, but good practice).