

Problem 1. Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9 - x}$$

(b)

$$\lim_{x \rightarrow -\infty} \frac{3x^3 + \sqrt[3]{x}}{\sqrt{9x^6 + 2x^2 + 1} + x}$$

(c)

$$\lim_{x \rightarrow 1} \frac{\sin^2(x - 1)}{(x - 1)^2} =$$

(d)

$$\lim_{x \rightarrow 3} \frac{x - 5}{(x - 3)^2} =$$

Problem 2.

(a) Using the Squeeze Theorem, show that

$$\lim_{x \rightarrow 3} \frac{x - 3}{1 + \sin^2\left(\frac{2\pi + e + 7}{x - 3}\right)} = 0.$$

(b) Let

$$g(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & x > 0 \\ x^2 + 1 & x < 0 \end{cases}$$

If possible, define an extension of g that is continuous at all real numbers.

Problem 3.

(a) **Directly from the definition of derivative**, compute the derivative of $f(x) = x^2 + \sqrt{x}$ at $a = 2$.

(b) **Naming each derivative rule used explicitly**, compute the derivative of $g(x) = x\sqrt{x^2 + 1}$.

Problem 4.

(a) Find an equation of the line tangent to $y = \frac{x^2-1}{x^2+1}$ at the point $(0, -1)$.

(b) Use a linear approximation to estimate $\sqrt{4.01}$.

(c) Give equation for the linear approximation of the function $f(x) = x \sin(x)$ near the point $a = \pi/2$.

Problem 5. Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) $f(x) = \sec\left(\frac{\sqrt{x^2+1}}{x+2}\right)$

(b) $g(x) = \sqrt[4]{\frac{x^3 + \cos(x^2)}{\sin(x^3) + 1}}$