

**Problem 1.**

(a) Let  $F(x) = 1/x + 1$  be the amount of pressure exerted on a beam in pounds per square inch at a point  $x$  inches to the right of its left end.

(i) What does the derivative  $F'(x)$  represent, and what are its units?

(ii) Compute  $F'(5)$ . What does this tell you?

(b) Suppose that  $Q(p) = 3p^2 + 10p - 100$  is the number of widgets you can buy at a price of  $p$  dollars.

(i) What does the derivative  $Q'(p)$  represent, and what are its units?

(ii) Calculate  $Q'(10)$ . What does this tell you?

(c) A radioactive substance begins decaying from 100g of material. When it reaches 10g, it is decaying at rate of 1g per year. After how many years does this occur?

**Problem 2.**

(a) Suppose a 100-liter tank contains a mixture of salt and water. Salt is added to the tank at a constant rate, and leaves the tank at a rate proportional to the concentration of salt in the tank. Write a differential equation to model this scenario, and explain in a couple sentences what your variables mean and why your equation is a good model.

(b) Check whether the function  $f(x) = xe^x$  is a solution to the differential equation  $y'' = 2y' - y$ . Justify your answer.

(c) Compute  $\frac{d}{dx}e^{\sin(x^3)+x}$ .

**Problem 3.**

(a) Suppose we have the differential equation  $f'(t) = f(t)(1 - f(t))$ , and  $f(0) = 1/2$ . Use Euler's method with three steps to approximate  $f(3)$ .

(b) Suppose we have the differential equation  $f'(t) = f(t) - t$ , with  $f(1) = 2$ . Use Euler's method with three steps to approximate  $f(4)$ .

**Problem 4.**

(a) Find a formula for  $y'$  in terms of  $x$  and  $y$  if  $x^8 + x^4 + y^4 + y^6 = 1$ .

(b) Find a tangent line to the curve given by  $x^4 - 2x^2y^2 + y^4 = 16$  at the point  $(\sqrt{5}, 1)$ .

(c) If  $x^2y = x + y$  find a formula for  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

**Problem 5.**

(a) The surface area of a cube is given by the formula  $A = 6s^2$  where  $s$  is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

(b) A car is driving down a road at 150 feet per second (this is about a hundred miles an hour). A camera is placed 200 feet from the road, which will rotate to follow and record the progress of the car. How quickly must the camera rotate when the car is fifty feet away from directly in front of the camera?