Math 322 Fall 2019
Number Theory HW 1
Due Friday, September 6

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem. Submit this problem on a separate, detached sheet of paper.

$\star$ **Redo Problem:** Show that if a, b, c, d are non-zero integers, and $a|b$ and $c|d$, then $ac|bd$.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Compute
   - $(6, 8, 10)$
   - $(34, 22)$
   - $(2970, 2925)$
2. Prove that if a and b are integers, then $\gcd(a, b)$ exists and is unique.
3. Let a, b, c be integers with $(a, b) = 1$ and $c|(a + b)$. Prove that $(a, c) = (b, c) = 1$.

   Definition 0.1. We say that a set of numbers $a_1, \ldots, a_n$ are *mutually relatively prime* if $\gcd(a_1, \ldots, a_n) = 1$. We say that the set is *pairwise relatively prime* if $\gcd(a_i, a_j) = 1$ whenever $i \neq j$.

4.
   - Find a triple of numbers (a, b, c) that are mutually relatively prime but not pairwise relatively prime.
   - Find a quadruple of numbers (a, b, c, d) that are mutually relatively prime, but any subset of three of them is not mutually relatively prime.

   Definition 0.2. If a and b are integers, then we define the *least common multiple* of a and b, written $[a, b]$ or $\text{lcm}(a, b)$, as the least (positive) integer c such that $a|c$ and $b|c$.

5. Let k be a natural number. Prove that $\text{lcm}(ka, kb) = k \cdot \text{lcm}(a, b)$.