

Math 322 Fall 2019
Number Theory HW 2
Due Friday, September 13

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem.

(★) **Starred Problem:** Prove that the greatest common divisor of two even numbers is even.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Let a and b be integers. For what integers c does the equation $ax + by = c$ have an integer solution? (Hint: pay attention to the form of the left hand side of this equation).
2. Use the Euclidean algorithm, showing all your steps, to compute $(94012, 33396)$. (Feel free to use a calculator).
3. If n is a positive integer, find (with proof) $(2n^2 + 6n - 4, 2n^2 + 4n - 3)$.
4. Let p be a prime and n an integer such that p does not divide n . Prove that $\gcd(p, n) = 1$.
5. Show that if c is an integer, then $\text{lcm}(a, b) | c$ if and only if $a | c$ and $b | c$. (Hint: use the division algorithm).
6. Let $p > 1$ be an integer with the following property: whenever a, b are integers and $p | ab$, then $p | a$ or $p | b$. Prove that p is prime.