

Math 322 Fall 2019
Number Theory HW 5
Due Friday, October 5

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem.

(★) **Starred Problem:** Suppose p is an odd prime and a is a positive integer with $(a, p) = 1$. Show that the congruence $x^2 \equiv a \pmod{p}$ either has no solution, or has exactly two solutions modulo p .

(Hint: assume a solution exists. Find another solution. Then prove that there are no more by assuming that y is any other solution and proving it's the one you found. This problem doesn't require anything sophisticated like Hensel's Lemma).

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Find the solutions of $x^2 + x + 34 \equiv 0 \pmod{81}$.
2. Find all solutions of $x^3 + 8x^2 - x - 1 \equiv 0 \pmod{1331}$. (Hint: $1331 = 11^3$).
3. Find all solutions to $x^5 + x - 6 \equiv 0 \pmod{144}$.