Math 322 Fall 2019
Number Theory HW 7
Due Friday, October 18

You may *not* discuss the starred problem with classmates, though you should of course feel free to discuss it with me as much as you like. Linguistic precision is important for this problem.

($\star$) **Starred Problem:** Show that if n is odd, then $\phi(4n) = 2\phi(n)$.

For the remainder of these problems, I encourage you to collaborate with your classmates, as well as to discuss them with me.

1. Let m be a natural number. Find a reduced residue system modulo 2^m.
2. Use Euler's theorem to find the last decimal digit of:
   (a) 3^{1000}
   (b) $7^{999,999}$
3. Let n be a natural number. Prove that $\phi(n) = n - 1$ if and only if n is prime.
4. Let a, b be relatively prime natural numbers. Show that $a^{\phi(b)} + b^{\phi(a)} \equiv 1 \mod ab$.
5. Let $c_1, c_2, \ldots, c_{\phi(m)}$ be a reduced residue system modulo m, where $m > 2$. Show that $c_1 + c_2 + \cdots + c_{\phi(m)} \equiv 0 \mod m$.
6. Determine whether each of the following functions is multiplicative, completely multiplicative, or neither.
   (a) $f(n) = 0$
   (b) $\gcd(n, k)$ for some fixed integer k.
   (c) $\log(n)$