

Math 114 Spring 2019
Calculus I HW 3 Solutions
Due Wednesday, February 13

1. Compute $\lim_{x \rightarrow -\infty} \frac{x^3 + 1}{\sqrt{x^6 + x^4 + 1}}$.

Solution:

$$\begin{aligned}\lim_{x \rightarrow -\infty} \frac{x^3 + 1}{\sqrt{x^6 + x^4 + 1}} &= \lim_{x \rightarrow -\infty} \frac{1 + 1/x^3}{\sqrt{x^6 + x^4 + 1}/(-\sqrt{x^6})} \\ &= \lim_{x \rightarrow -\infty} \frac{1 + 1/x^3}{-\sqrt{1 + 1/x^2 + 1/x^6}} \\ &= \frac{1 + 0}{-\sqrt{1 + 0 + 0}} = -1.\end{aligned}$$

2. Compute $\lim_{x \rightarrow +\infty} x^2 - x$.

Solution:

$$\begin{aligned}\lim_{x \rightarrow +\infty} x^2 - x &= \lim_{x \rightarrow +\infty} \frac{x^2 - x}{1} \\ &= \lim_{x \rightarrow +\infty} \frac{1 - 1/x}{1/x^2}.\end{aligned}$$

The limit of the top is 1 and the limit of the bottom is 0, so the limit is $\pm\infty$. Further, the bottom is always positive, so the limit is in fact $+\infty$.

3. Compute $\lim_{x \rightarrow +\infty} 2x - \sqrt{4x^2 + 3x + 1}$

Solution:

$$\begin{aligned}\lim_{x \rightarrow +\infty} 2x - \sqrt{4x^2 + 3x + 1} &= \lim_{x \rightarrow +\infty} \frac{4x^2 - (4x^2 + 3x + 1)}{2x + \sqrt{4x^2 + 3x + 1}} \\ &= \lim_{x \rightarrow +\infty} \frac{-3x - 1}{2x + \sqrt{4x^2 + 3x + 1}} \\ &= \lim_{x \rightarrow +\infty} \frac{-3 - 1/x}{2 + \sqrt{4 + 3/x + 1/x^2}} \\ &= \frac{-3 - 0}{2 + \sqrt{4 + 0 + 0}} = \frac{-3}{4}.\end{aligned}$$

4. If $f(x) = \sqrt[3]{x}$, then $f'(27) = \frac{1}{27}$. Use linear approximation to estimate $\sqrt[3]{25}$ and $\sqrt[3]{30}$.

Solution:

$$f(25) \approx f(27) + \frac{1}{27}(25 - 27) = 3 + \frac{1}{27}(-2) = \frac{79}{27} \approx 2.925$$

$$f(30) \approx f(27) + \frac{1}{27}(30 - 27) = 3 + \frac{1}{27}(3) = \frac{28}{9} \approx 3.\overline{11}.$$

5. Let $g(x) = x^2$. Use the definition of derivative to compute $g'(3)$ and then linearly approximate 3.2^2 .

Solution:

$$\begin{aligned} g'(3) &= \lim_{x \rightarrow 3} \frac{g(x) - g(3)}{x - 3} = \lim_{x \rightarrow 3} \frac{x^2 - 3^2}{x - 3} \\ &= \lim_{x \rightarrow 3} x + 3 = 6 \end{aligned}$$

$$g(3.2) \approx g(3) + 6(3.2 - 3) = 9 + 6 \cdot .2 = 10.2.$$

- 6. Stewart 2.1.26
- 7. Stewart 2.1.28
- 8. Stewart 2.1.30
- 9. Stewart 2.1.32
- 10. Stewart 2.1.34
- 11. Stewart 2.2.22
- 12. Stewart 2.2.24
- 13. Stewart 2.2.34
- 14. Stewart 2.2.36