

Math 114 Spring 2019
Calculus I HW 5 Solutions
Due Wednesday, March 6

For this homework you may compute derivatives using any tools we have developed in class.

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|-------------------|--------------------|
| 1. Stewart 2.1.10 | 6. Stewart 2.3.56 |
| 2. Stewart 2.1.12 | 7. Stewart 2.4.50 |
| 3. Stewart 2.1.46 | 8. Stewart 2.4.54 |
| 4. Stewart 2.1.48 | 9. Stewart 2.5.64 |
| 5. Stewart 2.3.52 | 10. Stewart 2.5.66 |

11. Verify that $f(x) = \sin(x^2)$ is a solution to the differential equation $y'' = y'/x - 4x^2y$.

Solution: We compute

$$\begin{aligned}f'(x) &= 2x \cos(x^2) \\f''(x) &= 2 \cos(x^2) - 4x^2 \sin(x^2)\end{aligned}$$

so

$$f''(x) = (2x \cos(x^2))/x - 4x^2(\sin(x^2)) = f'(x)/x - 4x^2 f(x).$$

12. Suppose $f(x) = ax^2 + bx + c$ satisfies $f(2) = 1$, $f'(2) = 2$, $f''(2) = 3$. Find $f(x)$.

Solution: We have $1 = f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$; we have $2 = f'(2) = 2a(2) + b = 4a + b$; and we have $3 = f''(2) = 2a$. This tells us that $a = 3/2$. Thus $2 = 6 + b$ so $b = -4$. Finally, we get $1 = 6 - 8 + c$ so $c = 3$. Thus $f(x) = 3x^2/2 - 4x + 3$.

13. Prove that $f(x) = \sin(x) + x^2$ satisfies $f''(x) + f(x) = x^2 + 2$.

Solution: We see that $f'(x) = \cos(x) + 2x$ and $f''(x) = -\sin(x) + 2$. Thus

$$f''(x) + f(x) = -\sin(x) + 2 + \sin(x) + x^2 = x^2 + 2$$

as desired.

14. Suppose the rate at which people hear about a piece of news is proportional to the number of people who have *not* heard about it. Write a differential equation to express this model, and identify the units of each variable and function you use.

Solution: Let $P(t)$ be the number of people who have heard the news at a time t . Then the input units are something like “days” and the output units are “people”. Let M be the total number of people (again, the units are “people”).

Then the number of people who have not heard the news at time t is $M - P(t)$, and so we get the differential equation

$$P'(t) = k(M - P(t)).$$

Alternatively, we could let $Q(t)$ be the number of people who have not heard the news; then we have

$$-Q'(t) = kQ(t).$$

15. Suppose the rate at which the concentration of a drug in the bloodstream decreases is proportional to the current concentration. Write a differential equation to express this model, and identify the units of each variable and function you use.

Solution:

Let $C(t)$ be the concentration of a drug in the bloodstream at the time t ; the units of $C(t)$ should be something like milligrams/liter, and the units of t are probably hours.

Then we get the differential equation

$$-C'(t) = kC(t).$$

The negative sign isn't strictly necessary; if k is positive then we need the minus sign, and if k is negative we do not. But the minus sign is probably a better choice because it makes it easier to see what is happening.

16. Suppose the rate at which an epidemic spreads through a community is jointly proportional to the number of people who have caught the disease and to the number of people who have not. Write a differential equation to express this model, and identify the units of each variable and function you use.

Solution: Let $D(t)$ be the number of people who have caught the disease, with t having units of days and $D(t)$ having units of people. Let M be the total number of people. Then we get the equation

$$D'(t) = kD(t)(M - D(t)).$$