Lab 10 Tuesday April 23

Quadratic Approximation

Exercises

For each exercise, plot the graphs of the true function, the linear approximation, and the quadratic approximation. Makes sure they match!

1. Use a quadratic approximation to estimate $\sqrt[3]{28}$.
2. If $f(x) = (x+8)^{1/2}$, compute a quadratic approximation centered at $x = 1$. Use this to estimate $f(1.02) = \sqrt{9.02}$.
3. If $f(x) = x^2 - 2x - 3$, compute a quadratic approximation centered at $x = 3$. How does this approximation compare to your original function?
4. Let $g(x) = x^4 - 3x^3 + 4x^2 + 4x - 2$. Compute the quadratic approximation at $x = -2$. Compare that to $g(x)$. Use this to estimate $g(-1.97)$.
5. Compute the quadratic approximations of $\sin(x)$ and $\cos(x)$ centered at zero. Estimate $\sin(.01)$ and $\cos(.01)$? How does this relate to the Small Angle Approximation?
6. Compute the quadratic approximation of e^x centered at 0. Estimate $e^{1/10}$ and $e = e^1$.
7. Compute the quadratic approximation of $\ln(1+x)$ centered at zero. Use this to estimate $\ln(1.1)$ and $\ln(2)$. How accurate do you expect these approximations to be? Check the true answers in Mathematica. Now try approximating $\ln(0)$.
8. If $f(x) = e^{x+x^2}$, find a formula for the quadratic approximation near zero, and use that to estimate $f(-.1)$.
9. Compute the quadratic approximation of $(1+x)^\alpha$ centered at 0. Use this formula to estimate 2^{10}. Use it to estimate 1.1^{10}.

Bonus: Special Relativity

Many formulas in the theory of special relativity depend on a parameter

$$\gamma(v) = \frac{1}{\sqrt{1-(v/c)^2}}$$

where v is the velocity, and c is the speed of light.

(a) What is $\gamma(0)$?

(b) Compute formulas for the linear and quadratic approximations to $\gamma(v)$ centered at zero. These tell us what happens when v is small relative to the speed of light.

(c) You are probably familiar with the famous formula that $E = mc^2$. This formula is for "rest energy", and holds when $v = 0$. For a moving object, we can compute the kinetic energy at a given velocity with the formula

$$E(v) = mc^2\gamma(v).$$

What happens if we replace γ with the quadrtic approximation? Does this look familiar?