Lab 7 Tuesday, March 19

Implict Functions and their Tangents

When using the `ContourPlot` command, note the double `==` signs.

1. (a) In class, we showed that the tangent line to $x^3+y^3 = 6xy$ at the point $(3,3)$ is $y-3 = 3-x$. Verify this with the command
`ContourPlot[{x^3 + y ^3 == 6 x*y, y-3 == 3 -x}, {x,-5,5},{y,-5,5}]`

   (b) We also derived the formula
$$y' = \frac{6y - 3x^2}{3y^2 - 6x}.$$
We see that the tangent line will be horizontal when $6y = 3x^2$ and also $x^3 + y^3 = 6xy$; doing some algebra shows that this happens at $(\sqrt[3]{16}, \sqrt[3]{32})$. Find an equation for the tangent line at this point, and plot it together with the graph of the curve.

2. Use `ContourPlot` to plot the graph of the curve `y Cos[x] = 1 + Sin[x*y]` that we studied in class yesterday. Can you guess what the tangent lines to this graph should look like?

3. (a) Use ContourPlot to plot the "cardioid" with equation:
`x^2 + y^2 == ( 2x^2 + 2y^2 -x)^2`. (x and y domains from -1 to 1).

   (b) Compute the derivative at the point $(0, 1/2)$ by hand.

   (c) Check your computation by running the commands
`D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x]` and
`D[x^2 + y[x]^2 == (2x^2 + 2y[x]^2 - x)^2,x] /. y[x] -> 1/2 /.x -> 0`

   Note some important details here. Mathematica can't figure out that `y` is a function of `x` instead of a constant unless we tell it, so we write `y[x]` instead of `y`. We can have Mathematica automatically substitute for us, but it matters that we do `/.y[x]->1/2` before `/.x->0`. Why? Try it the other way and see what happens.

   (d) Plot the tangent line to the cardioid at that point in Mathematica.

   (e) What do you expect to happen if you try to find the tangent line at $(0, 0)$? Are you right? What does Mathematica say?

   (f) Looking at the graph, what do you think is the tangent line at the point $(1, 0)$? Can you get this from your derivative formula? Try computing the (implicit) derivative with respect to y instead of x. What happens?

4. (a) Plot the "devil's curve" `y^2(y^2 -4) == x^2 (x^2 - 5)`

   (b) Compute the derivative at $(0, -2)$.

   (c) Plot the devil's curve and its tangent line simultaneously.

   (d) Run the command
`ContourPlot[y^2(y^2-4) - x^2(x^2 -5),{x,-5,5},{y,-5,5}]` What happens? Why?

5. (a) Plot `(x^2 + y^2 -1)^3 - x^2 * y^3 == 0`
   (b) Check that $(1, 1)$ is a solution to this equation, and compute the derivative at $(1, 1)$.
   (c) Plot the tangent line.
   (d) Now try plotting without the equals sign, as in (3).
6. (a) Plot `Sin[x^2 + y^2] == Cos[x * y]` from -5 to 5.
   (b) As before, replace the `==` with a `-` sign.

Just Because They're Pretty

1. Some other functions to try:
   - `Sin[Sin[x] + Cos[y]] == Cos[Sin[x * y] + Cos[y]]`
   - `Abs[Sin[x^2 - y^2]] == Sin[x + y] + Cos[x * y]`
   - `Csc[1-x^2] * Cot[2-y^2] == x * y`
   - `Abs[Sin[x^2 + 2 * x * y]] == Sin[x - 2 y]`
   - `(x^2 + y^2 - 3) Sqrt[x^2 + y^2] + .75 + Sin[4 Sqrt[x^2  + y^2]] Cos[84 ArcTan[y/x]] - Cos[6 ArcTan[y/x]] ==  0`
2. Try replacing the `==` signs with `-` signs.
3. Look at the examples on the Wolfram Alpha page
   `https://www.wolframalpha.com/examples/PopularCurves.html`