Lab 8 Tuesday April 2

Intermediate Value Theorem

1. Define a function `f[x_]:= x^4 - 8x^3 + 26x^2 - 35x +13`
   (a) Do you expect the equation $f(x) = 0$ to have any real solutions? Why or why not?
   (b) Compute `f[0]`, `f[1]`, `f[2]`, `f[3]`. Does this change your answer?
   (c) By guessing values, find an approximate root of f.
   (d) Use Mathematica to check your answers. Try the command `Solve[f[x]==0,x]` What happens? Why do you think that happens? Now try `NSolve[f[x]==0,x]` instead. What changes?
   (e) Run the command `Plot[f[x],{x,-1,3}]`. What do you see, and how does this relate to parts (b), (c), and (d)?
2. Define a function `g[x_]:= x^5-4x^2+1`.
   (a) Does g have a real root? Why or why not?
   (b) Try to find some real roots by plugging in values.
   (c) Graph g between -1 and 2 (with the command `Plot[g[x],{x,-1,2}]`). How many roots does it appear to have?
   (d) Use `Solve` and `NSolve` to check your answer.
3. Consider the function `Sec[x]` .
   (a) Calculate `N[Sec[0]]` and `N[Sec[1]]`. Is there some `c` where `Tan[c]` outputs 3/2? Why?
   (b) Now calculate `N[Sec[2]]`. Do you expect `Sec` to output 0 for some input between 1 and 2? (Think about this **before plotting the graph**).
   (c) Plot `Sec[x]` from 0 to 3. Does the graph match what you expected from part (b)?
4. Consider the piecewise function given by
   `a[x_] := Piecewise[{{x^2, x <= 0},{- x^2 - x^4, x >=0}}]`
   (a) Is `a[x]` a well-defined function? Is it continuous?
   (b) Compute `a[0]` and `a[1]`. Do you expect to find a solution to the equation `a[x] == -1`? Why or why not?
   (c) `Plot[a[x],{x,-1,1}]`
5. Now consider the piecewise function given by
   `b[x_]:=Piecewise[{{x^2, x <=0},{-2 - x^2 - x^4, x > 0}}]`
   (a) Is `b` well-defined as written? What did I have to change? Is it continuous?
   (b) Compute `b[0]` and `b[1]` Do you expect to find a solution to the equation `b[x] = -1`? Why or why not? What's different?
   (c) `Plot[b[x],{x,-1,1}]`

6. Define a function `c[x_] := x^4-6x^2-2x+2`
   (a) Plot it from -3 to 3. How many roots does it have, based on the graph?
   (b) Can you show that all of these have to exist using the Intermediate Value Theorem?

The IVT and solving equations

1. In class we looked at the function `f[x_] := x^3-x+1` and tried to solve the equation $f(x) = 4$.
   (a) Calculate `f[1]` and `f[2]`. What does this tell you about $f^{-1}(4)$?
   (b) Calculate `f[1.5]`. What does this tell you about $f^{-1}(4)$? Then calculate `f[1.75]`.
   (c) Keep going until you have estimated $f^{-1}(4)$ to two decimal places.
   (d) Use `NSolve` to check your answer.
2. Consider again the function `g[x_]:= x^5-4x^2+1`.
   (a) Calculate `g[-1]`, `g[0]` , `g[1]`, and `g[2]`. What does this tell you about the roots of g?
   (b) Calculate `g[1.5]`. Now should we check `g[1.25]` or `g[1.75]`?
   (c) Estimate the root of g in $(1, 2)$ to two decimal places.
   (d) Estimate the root of g in $(-1, 0)$ to two decimal places. Does this match the answer you got at the beginning of the sheet?
3. Use the Intermediate Value Theorem to find a c such that $\sin(c) \approx 1/3$.
4. Use the Intermediate Value Theorem to approximate $\sqrt[4]{3}$.
5. Use the Intermediate Value Theorem to find a c such that $\exp(c) = e^c \approx 2$.