Math 214 Spring 2019
Linear Algebra HW 4
Due Friday, March 1

1. (a) Write $x + x^2 + x^3$ as a linear combination of $x, x + 2x^2, x^2 - 4x^3$.
   (b) Write $4x + 6x^3 - x^5$ as a linear combination of $x + x^3, x^3 + x^5$, and $x + x^5$.

2. Let $V = \mathbb{R}^3$.
   (a) Is $S = \{(1, 2, 3), (2, 3, 4), (3, 4, 5)\}$ a spanning set for $\mathbb{R}^3$?
   (b) Is $T = \{(1, 2, 3), (2, 3, 4), (0, 1, 1)\}$ a spanning set for $\mathbb{R}^3$?

3. (a) Is $S = \{(1, 1, 0, 0), (1, -1, 0, 0), (0, 0, 1, -1), (0, 0, -1, 1)\}$ a spanning set for $\mathbb{R}^4$?
   (b) Is $T = \{1, 1 + x, 1 + x^2\}$ a spanning set for $\mathcal{P}_2(x)$?

4. Suppose $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\} \subset V$ is a spanning set for V. Prove that $T = \{\mathbf{v}_1, \mathbf{v}_2 - \mathbf{v}_1, \mathbf{v}_3 - \mathbf{v}_2, \ldots, \mathbf{v}_n - \mathbf{v}_{n-1}\}$ is a spanning set for V.

5. (a) Is $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ a linearly independent set?
   (b) Is $T = \{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$ a linearly independent set?
   (c) Is $U = \{(3, 7, 5), (2, 4, 2), (1, 3, 1)\}$ a linearly independent set?

6. (a) Is $S = \{1 + x, 1 + x^2, x + x^2\}$ a linearly independent set?
   (b) Is $T = \{1 + x, 1 + x^2, x - x^2\}$ a linearly independent set?
   (c) Is $U = \{\sin^2, \cos^2, 1\}$ a linearly independent set?

7. ($\star$) Suppose $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is linearly independent in V, and $T = \{\mathbf{v}_1 + \mathbf{w}, \ldots, \mathbf{v}_n + \mathbf{w}\}$ is linearly dependent in V. Prove that $\mathbf{w} \in \text{Span}(S)$.

8. Prove that a set $S = \{\mathbf{u}, \mathbf{v}\}$ of two vectors is linearly dependent if and only if one is a scalar multiple of the other.