

Math 214 Spring 2019
Linear Algebra HW 7
Due Friday, March 29

For all these problems, justify your answers; do not just write “yes” or “no”.

1. Let $A = \begin{bmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{bmatrix}$. Find bases for the row space, column space, and nullspace of A .

2. Let $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix}$. Find bases for the row space, column space, and nullspace of B .

3. Use Gaussian elimination to find a basis for the span of $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\}$.

4. For each of the following systems of equations, is there a solution? You don't need to find the solution if it exists, but justify your answer. (Hint: think about the column space).

(a)

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 0 & 0 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ?$$

(b)

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 1 \end{bmatrix} ?$$

5. (a) Find a basis for the image of the linear transformation $L(x, y, z) = \begin{bmatrix} x + y + z \\ 3x - 2y + z \\ 2z \end{bmatrix}$ that we saw in Homework 6 problem 3.
- (b) Find a basis for the image of the operator $P_Z(x, y, z) = (x, y, 0)$ that we saw in Homework 6 Problem 5.
6. (★) Let $U = \mathcal{P}_3(x)$, and define a linear map $D : U \rightarrow U$ by $D(f(x)) = f'(x)$. Let $E = \{1, x, x^2, x^3\}$ be a basis for U .

- (a) Describe the kernel and image of D .
 - (b) Find a matrix for D with respect to E (as a basis for both the domain and the codomain).
 - (c) Find bases for the kernel and image of D .
7. (★) Let $L : \mathcal{P}_3(x) \rightarrow \mathbb{R}^2$ be given by $L(f(x)) = (f(1), f(2))$, and let $E = \{1, x, x^2, x^3\}$ and $F = \{(1, 0), (0, 1)\}$.
- (a) Prove that L is a linear transformation.
 - (b) Find a matrix with respect to the bases E and F .
 - (c) Find bases for the kernel and image of L .