# Math 214 Test 1
# Practice Problems

Instructor: Jay Daigle

This is not a practice test, in the sense that it is not the format I expect the test to be. It is a collection of practice problems. I will update you when I finalize the test format.

## Solve the following systems of linear equations

1.

$$\begin{aligned} x - 4y + 2z &= 2 \\ -x + 3y + z &= 4 \\ 2x - y + z &= 1 \end{aligned}$$

2.

$$\begin{aligned} x + 3y + 7z &= 15 \\ 2x + 9y + 23z &= 45 \\ x - z &= 2 \end{aligned}$$

3.

$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ x_1 - 5x_2 + x_4 &= 5 \end{aligned}$$

4.

$$\begin{aligned} -x_1 + 2x_2 - x_3 &= 2 \\ -2x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + 2x_2 + 2x_3 &= 5 \\ -3x_1 + 8x_2 + 5x_3 &= 17 \end{aligned}$$

5.

$$\begin{aligned} x - 2y &= 3 \\ 2x + y &= 1 \\ -5x + 8y &= 4 \end{aligned}$$

## Do the following matrix multiplication computations.

1.

$$\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} =$$

2.

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \end{bmatrix} =$$

3.

$$\begin{bmatrix} 5 & 2 & -1 \\ 1 & 0 & 3 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} -3 & 0 & 1 \\ 4 & -1 & 1 \\ 0 & 2 & 3 \end{bmatrix} =$$

4.

$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 1 \\ 2 & 3 & 8 \end{bmatrix} =$$

## For each of the following matrices, find:

(a) The reduced row echelon form.

(b) The nullspace

1. $\begin{bmatrix} 3 & 1 & 2 \\ -1 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} -2 & 4 & 1 \\ -5 & 1 & 1 \\ 3 & 3 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 6 & 2 & 3 & 1 \\ 1 & 5 & 2 & -2 \\ 4 & -4 & 1 & 3 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 3 & 4 \\ 2 & 5 & 2 \\ 0 & 1 & 3 \\ 4 & 1 & -2 \end{bmatrix}$

## Find the inverses of the following matrices, or show they are not invertible.

1. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 2 & -1 \\ 1 & -4 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$

## Which of the following are vector spaces? Prove or disprove your answer, potentially using the subspace theorem

1. $\{(a, b, c, d) : a - b = c - d\}$
2. $\{(a, b, c, d) : a + b + c = d\}$
3. $\{(a, b, c) : a^2 = bc\}$
4. $\{(a, b, c, d) : 5a - 3b = 2c - 2d\}$
5. $\{a_0 + a_1x + a_2x^2 + a_3x^3 : a_2 = 2\}$
6. $\{f(x) : f(0) = 5\}$
7. $\{f(x) : f(5) = 0\}$

## Write u as a linear combination of vectors in $S$, or prove you cannot

1. $\mathbf{u} = (5, 2, 1)$, $S = \{(1, 2, 3), (3, 1, 1)\}$
2. $\mathbf{u} = (2, 3, 2)$, $S = \{(1, 2, 3), (3, 4, 1)\}$
3. $\mathbf{u} = x^3 - x + 1$, $S = \{1 + x, 3 + x^2, 3x^2 + x^3\}$
4. $\mathbf{u} = x^3 + 4x^2 + 2x + 5$, $S = \{1 + x, 3 + x^2, 3x^2 + x^3\}$

## Proofs

1. Suppose $U, W$ are subspaces of some vector space $V$. Prove that the set $U + W = \{\mathbf{u} + \mathbf{w} : \mathbf{u} \in U, \mathbf{w} \in W\}$ is a subspace of $V$.

   Bonus: what is the space $U + U$?

2. Let $A \in M_{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$. Prove that if $N(A) = \{\mathbf{0}\}$ then the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution.

## Bonus to stretch your brain

1. Find a subset $U \subset \mathbb{R}^2$ that is closed under scalar multiplication but is not a subspace.
2. Find a subset $U \subset \mathbb{R}^2$ that is closed under addition but is not a subspace.