MULTIVARIABLE CALCULUS (*MATH 212*) COMMON TOPIC LIST
(Approved by the Occidental College Math Department on August 28, 2009)

Required Topics

- Multivariable functions
- 3-D space
    - Distance
    - Equations of Spheres
    - Graphing
        - Planes
        - Spheres
        - Miscellaneous function graphs
        - Sections and level curves (contour diagrams)
        - Level surfaces

- Planes
    - Equations of planes, in different forms
    - Equation of plane from three points
    - Linear approximation and tangent planes
- Partial Derivatives
    - Compute using the definition of partial derivative
    - Compute using differentiation rules
    - Interpret
    - Approximate, given contour diagram, table of values, or sketch of sections
    - Signs from real-world description
    - Compute a gradient vector
- Vectors
    - Arithmetic on vectors, pictorially and by components
    - Dot product
        - compute using components
        - $\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$
        - $\vec{u} \cdot \vec{v} = 0 \Leftrightarrow \vec{u} \perp \vec{v}$ (for nonzero vectors)
    - Cross product
        - Direction and length
        - compute using components and determinant
- Directional derivatives
    - estimate from contour diagram
    - compute using the $\lim_{t \to 0}$ definition
    - compute using $f_{\vec{u}} = \nabla f \cdot \vec{u}$ ($\vec{u}$ a unit vector)
- Gradient
    - geometry (3 facts, and understand how they follow from $f_{\vec{u}} = \nabla f \cdot \vec{u}$)
        - Gradient points in direction of fastest increase
        - Length of gradient is the directional derivative in that direction
        - Gradient is perpendicular to level set

    - given contour diagram, draw a gradient vector
- Chain rules
- Higher-order partials
    - compute
    - mixed partials are equal (under certain conditions)
- Optimization
    - Locate and classify critical points in a contour diagram
    - Find critical points given a formula for f
    - max-min (including word problems)
    - Lagrange multipliers
        - Understand basically why setting $\nabla f = \lambda \nabla g$ finds "candidates"
    - Extreme value theorem, including understanding what "closed" and "bounded" mean
    - Optimization on the boundary of a region by substitution
- Integration
    - Predict the sign of a multiple integral
    - Compute a multiple integral
    - Sketch the region of integration
    - Choose or change the order of integration
    - Some other coordinate system (e.g., polar, cylindrical, spherical), including the observation that this is a change of variables, similar to u-substitution from Calc 2
- Parametrized curves
    - Construct parametrizations of lines, circles, and explicitly defined curves, that move in a specified direction
    - Velocity and speed
- Vector fields
    - Sketch a vector field with a given formula
    - Recognize a gradient (conservative) field, and find a formula for a potential function
- Line Integrals
    - For a constant vector field $\vec{F}$ and a straight path $\vec{C}$, $work = \vec{F} \cdot \vec{C}$
    - Given a picture of a vector field, predict the sign of a line integral, or draw a path over which a line integral will have a given sign
    - Compute $\int_C \vec{F} \cdot d\vec{r}$ using $\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) dt$
    - For a gradient field, compute using the fundamental theorem of line integrals
    - For a gradient field, compute using path independence (parametrize a simpler path with the same endpoints)
    - For a gradient field, the line integral over a loop is zero
    - Green's theorem

Some Optional Topics

- Boats and planes (the kind that fly) and forces and vectors
- Prove something using vectors
- Projection
- Find plane tangent to implicitly defined surface by viewing surface as level set
- Implicit differentiation

- Signs of second partials from contour diagram
- Second derivative test
- Use limits to discuss the existence of global maxes/mins
- Polar area
- Polar volume
- Cylindrical coordinates and triple integrals
- Spherical coordinates and triple integrals
- The cool trick for computing the area under a bell curve
- Intersections of a curve with a surface
- Collisions and intersections of parametrized curves
- Distances, etc., involving parametrized lines
- Find a parametrization for a complicated curve by summing parametrizations of simple component motions
- Give a plausible formula for a vector field whose sketch is given
- Stokes' Theorem
- Divergence and curl
- Divergence Theorem