$f(x) = x^2$

$1 \mapsto 1$

$3 \mapsto 9$

Vending machine

• $\mapsto$ coke

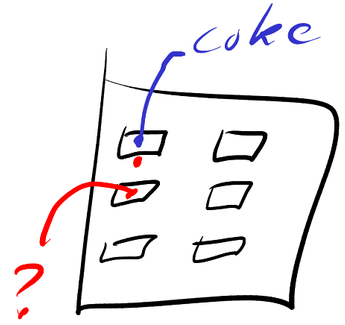
• $\mapsto$ sprite

Q: what is $\sqrt{4}$? 2

Q: what is $\sqrt{5}$? about 2

$\sqrt{4.1}$ really close to 2

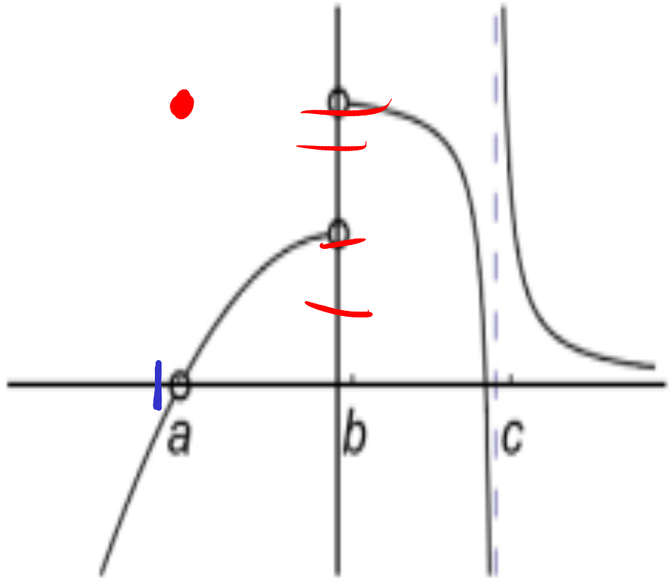
$\sqrt{4.01}$ super close to 2



$(3.1)^3 \approx 27$

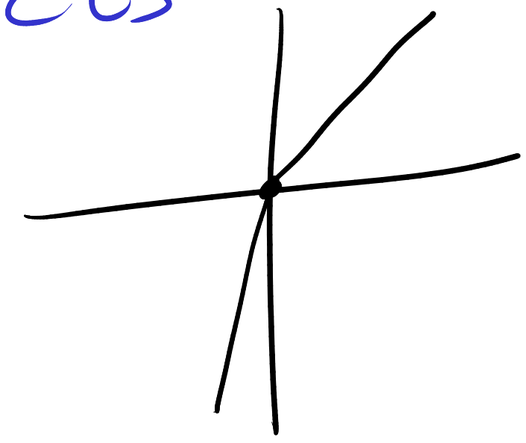
$\sqrt[3]{28} \approx 3$

$(3.00127)^3 \approx 27$

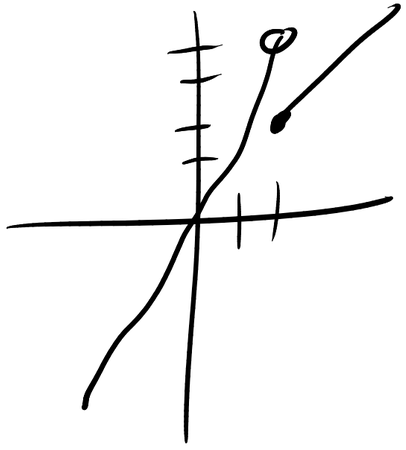


$$f_1(x) = \begin{cases} x & \geq 0 \\ 2x & < 0 \end{cases}$$

cts



$$f_2(x) = \begin{cases} x & \geq 2 \\ 2x & < 2 \end{cases}$$



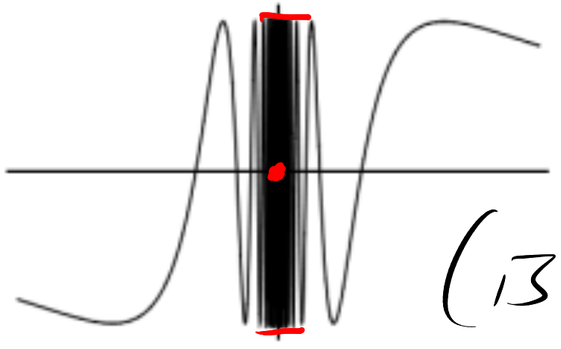
not cts

discontinuity @ 2

(jump)

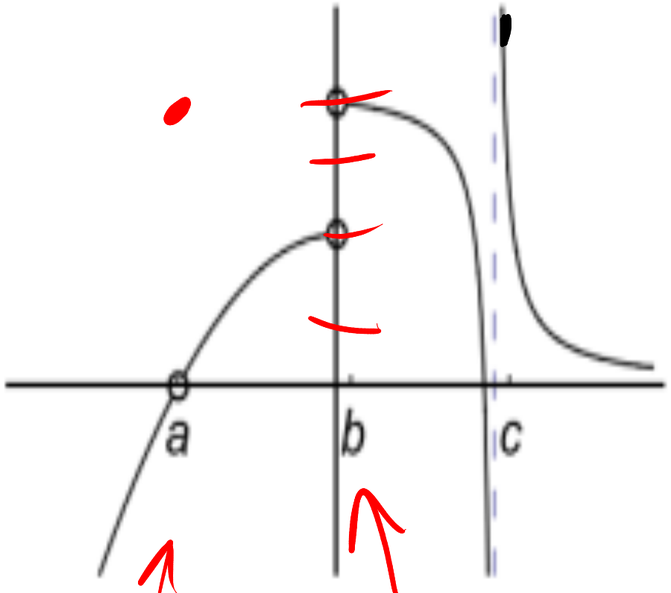
$$g(x) = \begin{cases} \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$g(.001) \approx ?$



Not cts

(is a Darboux fn)



"should" be 0, is not

2 or 4 jump

"$f(c) = \infty$"

∞ is not a number

"$\infty - 1 = \infty$"?

"$-1 = 0$"?

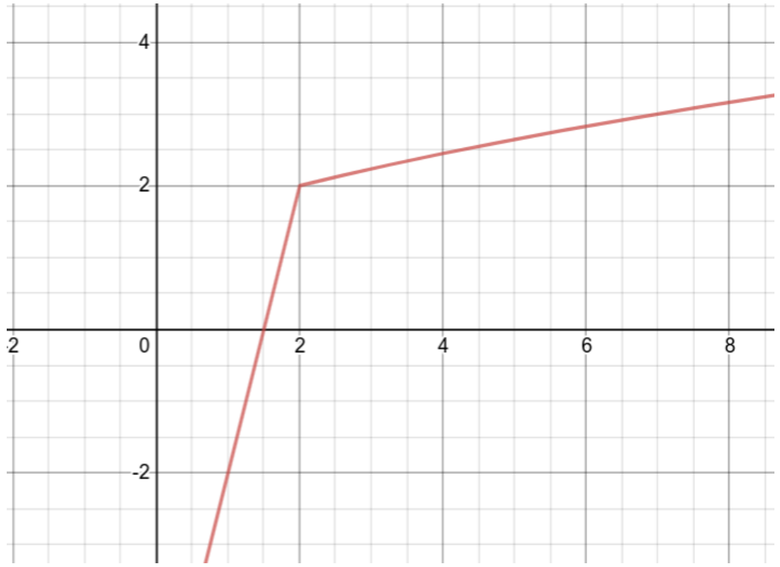
if $f(c) = \infty$

then $f(c+.001)$ is "about ∞"?

$$h(x) = \begin{cases} \sqrt{x+2} & x \geq 2 \\ 4x+b & x < 2 \end{cases}$$

$h(2) = 2$, $b = -6$

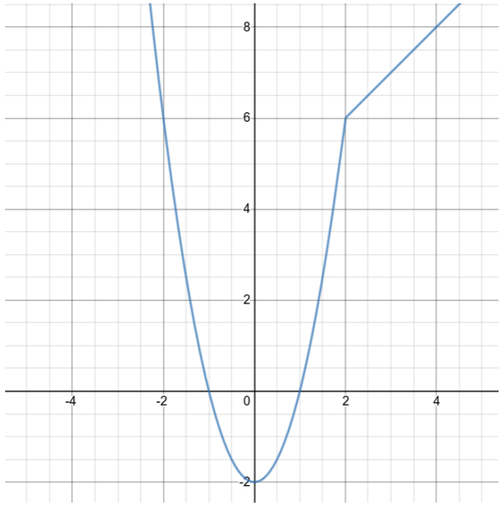
for what b is this cts, if any?



$$j(x) = \begin{cases} x+4 & x \geq 2 \\ ax^2-2 & x < 2 \end{cases}$$

$j(2) = 6$

$a = 2$.



Limit

$\lim_{x \to a} f(x) = L$

if

we can make f as close to L as we want by making x close to a.

Cts

Know $f(a)$

want to know $f(x)$ for x near a.

Limits

know $f(x)$ for x near a

Want: what $f(a)$ should be.

Speed

$\frac{\text{distance}}{\text{time}}$

$\frac{f(1) - f(0)}{1 - 0}$ $\frac{\text{miles}}{\text{hours}}$

Speed at noon

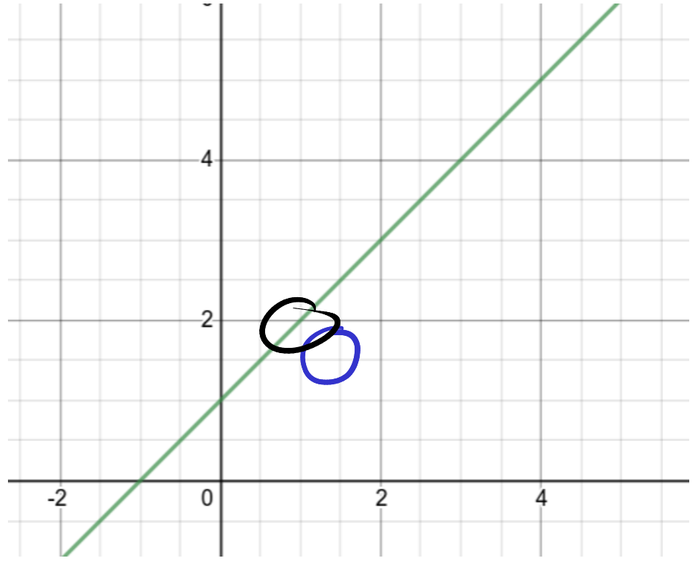
$\frac{f(0) - f(0)}{0 - 0} = \frac{0}{0}$ "!!" ☹

$\frac{f(.001) - f(0)}{.001 - 0} \approx$

$$\lim_{x \to 1} x+1 \overset{?}{=} 2$$ ✓

if cts and defined:

easy.



$$\frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1}$$

$$\neq x+1$$

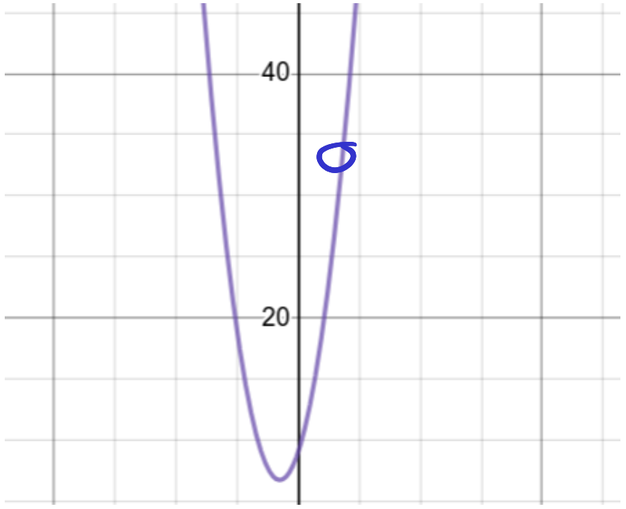
Almost identical

$$\lim_{x \to 1} \frac{x^2-1}{x-1} \to \frac{0}{0} = \lim_{x \to 1} x+1 = 2.$$

$$\frac{0}{0} = \text{"}\infty\text{"}$$

A) $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$

$= \lim_{x \to 3} (x^2 + 3x + 9) = 27$

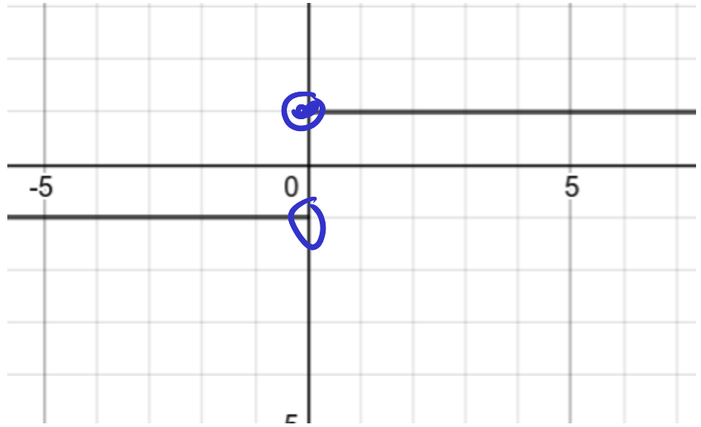


$\frac{x^3 - 27}{x - 3} = \frac{(x-3)(x^2+3x+9)}{x-3}$

$\neq x^2 + 3x + 9$

B) $\lim_{x \to 0} \frac{|x|}{x}$ (and sketch)

$\lim_{x \to 1} \frac{|x|}{x} = 1$



$x^3 - a^3 = (x-a)(x^2 + xa + a^2)$

$x^2 - a^2 \quad (x-a)(x+a)$

$$x-3 \overline{)\, x^3+0x^2+0x-27}$$

x^2

$x^3 - 3x^2$

$-3x^2+0x-27$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$\sin(x) \to 0$, $x \to 0$

$$\lim_{x\to 0} \frac{\cos x}{x} = \infty$$

$\cos x \to 1$, $x \to 0$