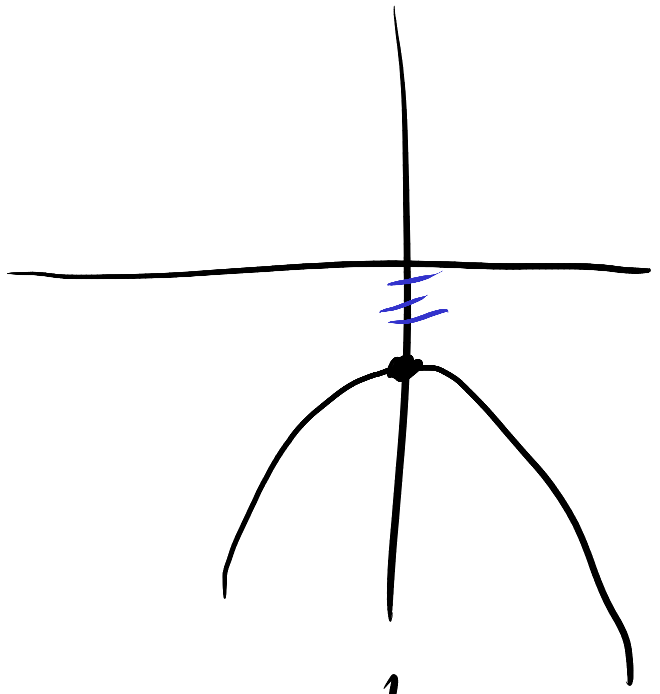
Continuity

Limits

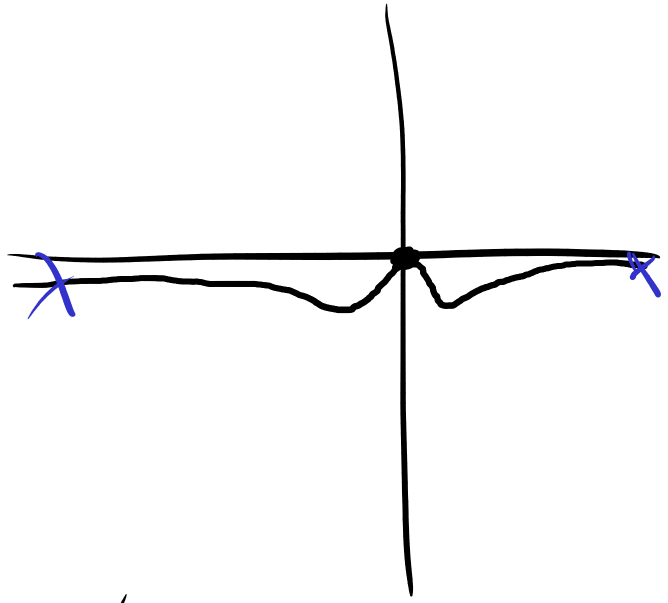
$$\lim_{x \to a} f(x) = L$$

Dfn: we say $\lim_{x \to a} f(x) = L$ if:

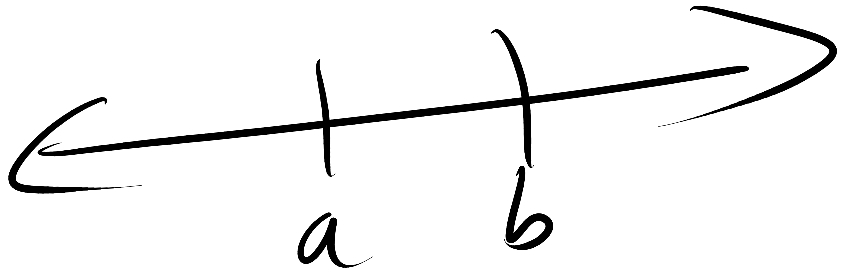
for every $\varepsilon > 0$,
there is a $\delta > 0$, so that if
$0 < |x-a| < \delta$
then $|f(x) - L| < \varepsilon$.



this limit is not zero



This limit is 0.



$\Delta = D$ $\delta = d$

$E = E$ $\varepsilon = e$

how far away
y values are
"error"

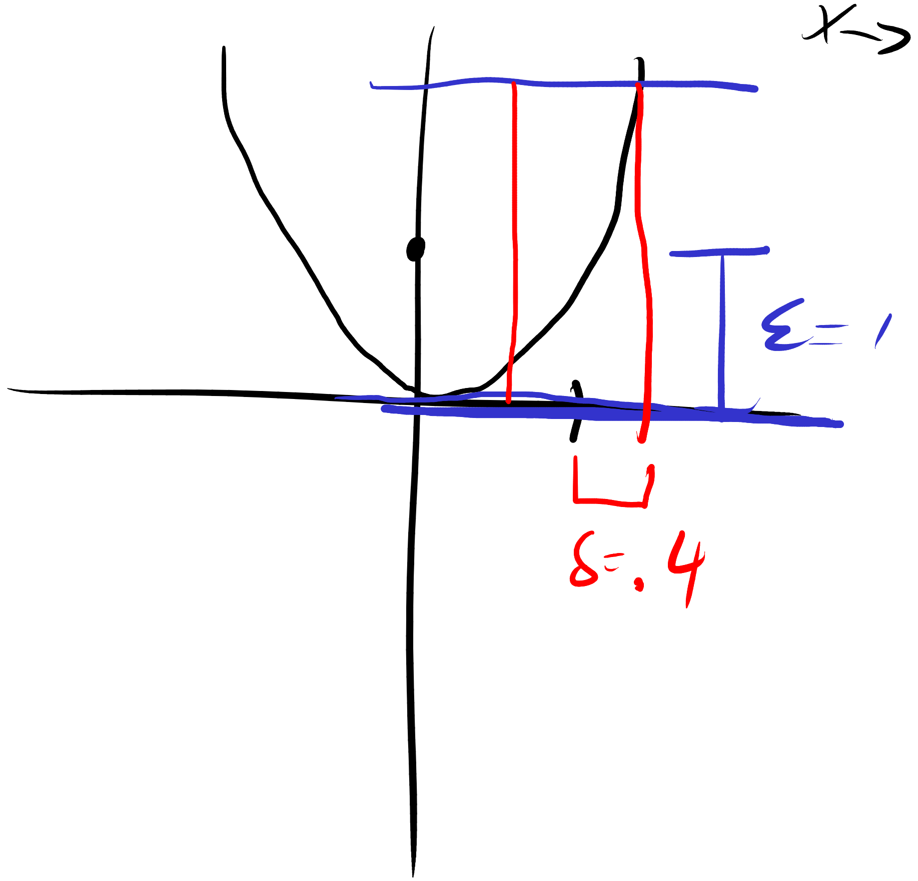
Dfn: we say $\lim_{x \to a} f(x) = L$ if:

for every $\boxed{\varepsilon > 0,}$
there is a $\boxed{\delta > 0,}$ so that if
$0 < |x-a| < \delta$
then $|f(x)-L| < \varepsilon$.

how far away
x values are
"distance"

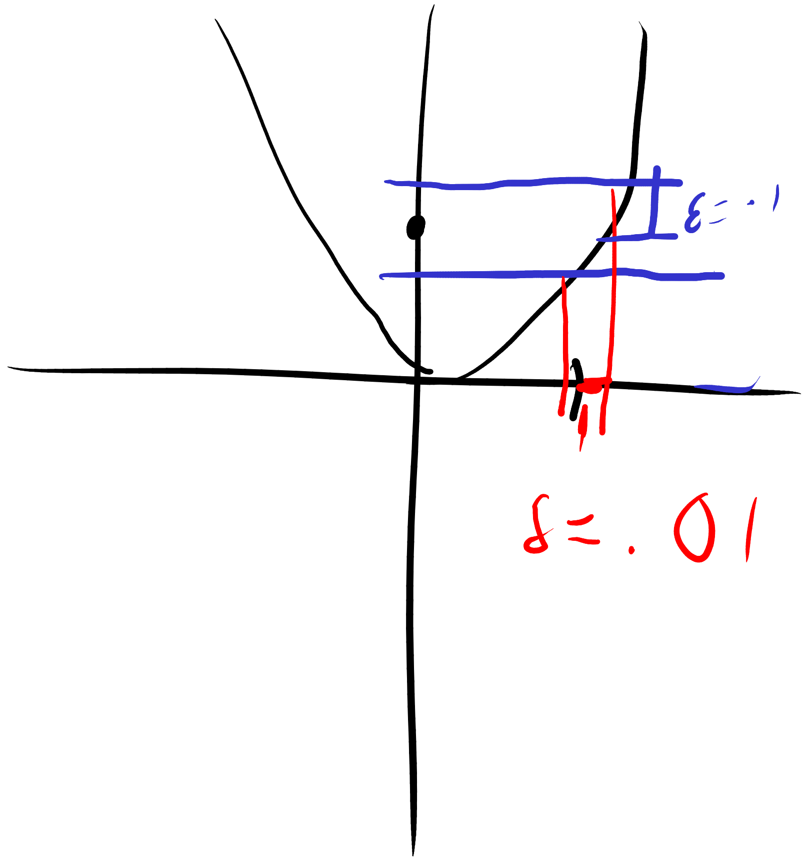
$f(x) = x^2$

$\lim_{x \to 1} x^2 = 1$



if I want $0 < x^2 < 2$
need $0 < x < \sqrt{2}$

if $0 < |x-1| < .4$
then $|x^2-1| < 1$



if $0 < |x-1| < .01$ $\quad .99 < x < 1.01$

then $|x^2-1| < .1$ $\quad .9 < x^2 < 1.1$

$f(x) = 2x+1$

$\lim_{x \to 3} 2x+1 \overset{?}{=} 7$

want $|2x+1-7|<1$

if $\varepsilon = 1$ want $6<2x+1<8$

$5<2x<7$

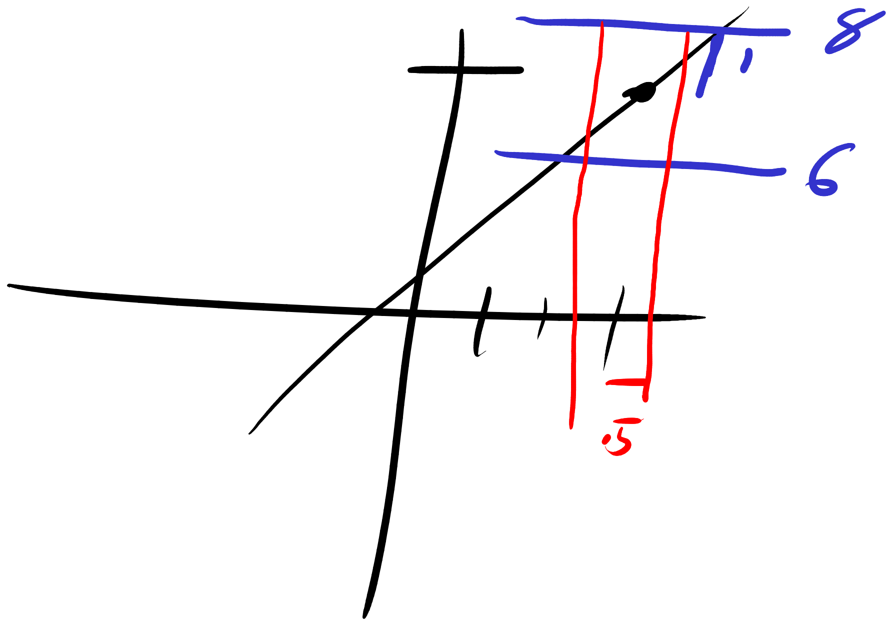
$\delta = .5$ $\quad 2.5<x<3.5$

$\quad\quad '' \quad\quad\quad ''$

$3-.5 \quad\quad 3+.5$

$\varepsilon = .01$ $\quad\quad \delta = \varepsilon/2$

$\delta = .005$



If $\varepsilon = .1$ want $6.9<2x+1<7.1$

$\delta = .05$

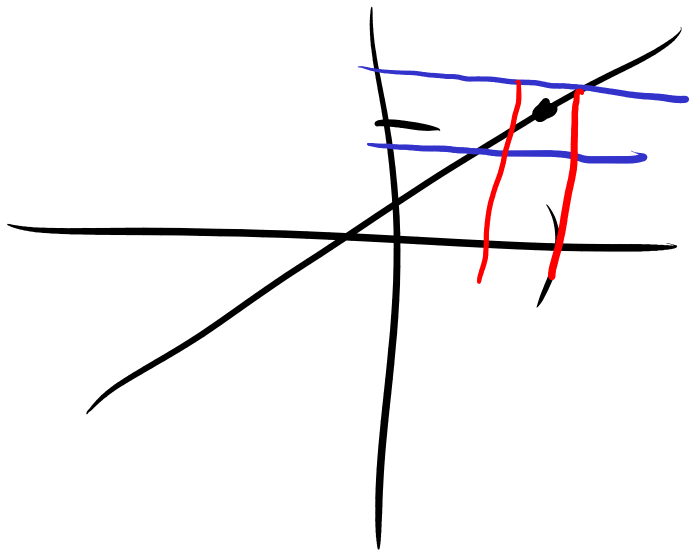
$5.9<2x<6.1$

$2.95<x<3.05$

$3-.05 \quad\quad 3+.05$

$f(x) = 2x+1$

$\lim_{x \to 3} 2x+1 \stackrel{?}{=} 7$



Proof: Let $\varepsilon > 0$. Set $\boxed{\delta = \varepsilon/2.}$

Suppose $0 < |x-3| < \delta$

$0 < |x-a| < \delta$

Then $|f(x)-L| = |2x+1-7| = |2x-6| = 2|x-3| < \boxed{2\delta = 2\frac{\varepsilon}{2}} = \varepsilon.$

for every $\varepsilon > 0$, there is a $\delta > 0$, so that if

$0 < |x-a| < \delta$

then $|f(x)-L| < \varepsilon$

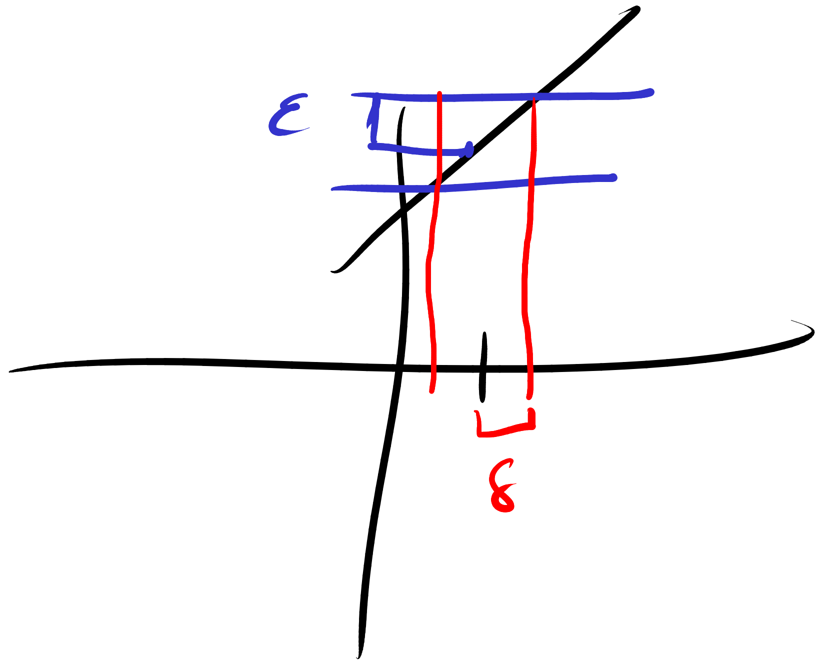
$|f(x)-L| < 2\delta$

want $2\delta = \varepsilon$

$2\delta = \varepsilon \Rightarrow \delta = \varepsilon/2$

$f(x) = 3x+5$

$\lim_{x\to 1} 3x+5 = 8$



Pf/ Let $\varepsilon > 0$. Set $\delta = \underline{\varepsilon/3}$.

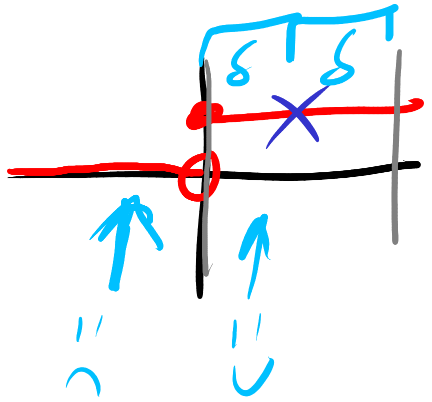
Then if $0 < |x-1| < \delta$

Then $|f(x) - L| = |3x+5-8| = |3x-3| = 3|x-1| < 3\delta = 3\frac{\varepsilon}{3} = \varepsilon$.

Want $3\delta = \varepsilon$

$\delta = \varepsilon/3$

$$H(x)=\begin{cases}1 & x\geq 0\\ 0 & x<0\end{cases}$$



$|x|<3$

$-3<x<3$

$|-4|$

$$\lim_{x\to 1} H(x)=1$$

Let $\varepsilon>0$. Set $\delta=1$

Then if $0<|x-1|<1$ this is $0<x<2$

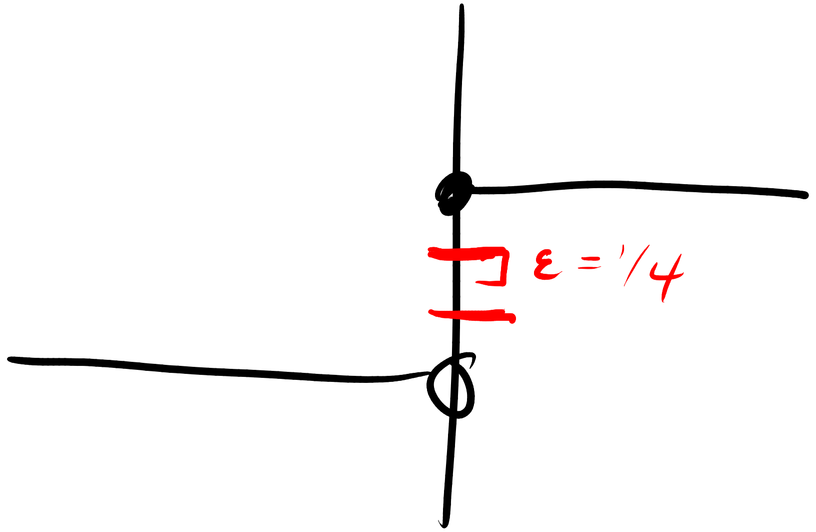
Then $||H(x)-1||=|1-1|=0<\varepsilon$.

$|x-1|<1$

$-1<x-1<1$

$0<x<2$

$$\lim_{x\to 0} H(x) \text{ DNE}$$



Limit Laws

Lemma (Identity): $\lim_{x\to a} x = a$

Let $\varepsilon > 0$. Set $\delta = \varepsilon$

If $0 < |x-a| < \delta$,

then $|f(x) - L| = |x - a| < \delta = \varepsilon$.

(constants)

$\lim_{x\to a} c = c$

$$\lim_{x\to 3} \frac{x^2+5}{x-3}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} x+1$$

(AIF)

Almost Identical functions