



$$x^2 + y^2 = 25$$

$$\frac{d}{dy} x^2 + y^2 = \frac{d}{dy} 25$$

$$2x \frac{d}{dy} x + 2y \frac{dy}{dy} = 0$$

$$2x x' + 2y = 0$$

$$2x x' = -2y$$

$$x' = -y/x$$

$$x' = -4/3$$

$$x \approx x_0 + x'(y - y_0)$$

$$\text{T line @ } -3, 4$$

$$y' = -x/y = 3/4$$

$$y = 4 + \frac{3}{4}(x + 3)$$

$$\text{@ } 3, -4$$

$$y' = -x/y = 3/4$$

$$y = -4 + \frac{3}{4}(x - 3)$$

$$\text{@ } 0, 5$$

$$y' = -x/y = 0$$

~~$$\text{@ } 5, 0$$~~

~~$$y' = -x/y \quad \ddot{\sim}$$~~

$$x' = -y/x = 0$$

$$x \approx 5 + 0(y - 0)$$

$$x^3 + y^3 = 6xy$$

T line @ (3, 3)

$$y = 3 - 1(x - 3)$$

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx} 6xy$$

$$3x^2 + 3y^2 \cdot y' = 6(1 \cdot y + y'x)$$

$$3y^2 \cdot y' - 6xy' = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

$$y'(3, 3) = \frac{18 - 27}{27 - 18} = \frac{-9}{9} = -1$$

$$27 + 27y' = 6(3 + 3y')$$

$$27 + 27y' = 18 + 18y'$$

$$27 + 9y' = 18$$

$$9y' = -9 \quad \text{so } y' = -1$$

$$y'' = \frac{d}{dx} \frac{2y - x^2}{y^2 - 2x}$$

$$= \frac{(2y' - 2x)(y^2 - 2x) - (2yy' - 2)(2y - x^2)}{(y^2 - 2x)^2}$$

$$= \frac{\left(2 \frac{2y - x^2}{y^2 - 2x} - 2x\right)(y^2 - 2x) - \left(2y \frac{2y - x^2}{y^2 - 2x} - 2\right)(2y - x^2)}{(y^2 - 2x)^2}$$

$$y \cos(x) = 1 + \sin(xy)$$

$$- \sin(x) \cancel{y}$$

$$\cancel{y} + y'x$$

$$y' \cos(x) + (-\sin(x))y = \cos(xy) (1 \cdot y + y'x)$$

$$y' \cos(x) = y \cos(xy) + y'x \cos(xy) + y \sin(x)$$

$$y' \cos(x) - y'x \cos(xy) = y \cos(xy) + y \sin(x)$$

$$y' = \frac{y \cos(xy) + y \sin(x)}{\cos(x) + x \cos(xy)}$$

$$9x^2 + y^2 = 9$$

$$\textcircled{Q} (0, 3)$$

$$y''(0, 3) = -9 \frac{9 + 0}{27} = -3.$$

$$18x + 2yy' = 0$$

$$2yy' = -18x$$

$$y' = -9x/y \quad y'(0, 3) = \frac{-9 \cdot 0}{3}$$

$$y = 3 + 0(x - 0)$$

$$y'' = \frac{d}{dx} \left( -\frac{9x}{y} \right) = -9 \frac{d}{dx} \frac{x}{y}$$

$$= -9 \frac{1 \cdot y - y'x}{y^2} = -9 \frac{y + \frac{9x}{y}x}{y^2} = -9 \frac{y^2 + 9x^2}{y^3}$$

Suppose have  $f$  in  $f$

$$8 f(x) + x^2 f(x)^3 = 24$$

Know  $f(4) = 1$ .

estimate  $f(5)$ ?

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$f(5) \approx f(4) + f'(4)(5-4)$$

$$= 1 + \frac{-1}{2} \cdot 1 = \frac{1}{2} \approx .5$$

Check:  $f(5) \approx .879$

$$8 f'(x) + 2x f(x)^3 + 3 f(x)^2 f'(x) x^2 = 0$$

$$8 f'(4) + 2 \cdot 4 \cdot f(4)^3 + 3 f(4)^2 f'(4) \cdot 16 = 0$$

$$8 f'(4) + 8 + 48 f'(4) = 0$$

$$f'(4) + 1 + 6 f'(4) = 0$$

$$\Rightarrow f'(4) = -1 \quad f'(4) = -1/7.$$