$f(x) = 2x^3 + 3x^2 - 36x$

$f'(x) = 6x^2 + 6x - 36$

$= 6(x^2 + x - 6) = 6(x+3)(x-2)$

CP: $-3, 2$

$f''(x) = 12x + 6$

$f''(-3) = -30 < 0$

so we have a max at -3.

$f''(2) = 30 > 0$

f is CU, we have a min at 2

if $f'' > 0$, f CU, min

if $f'' < 0$, f CD, max

$\cup$ CU

$\cap$ CD

$$f(x) = x^{2/3}(6-x)^{1/3}$$

$$f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$$

$$f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}$$

CP: 4, 0, 6

$$f''(4) = \frac{-8}{4^{4/3} \cdot 2^{5/3}} < 0$$

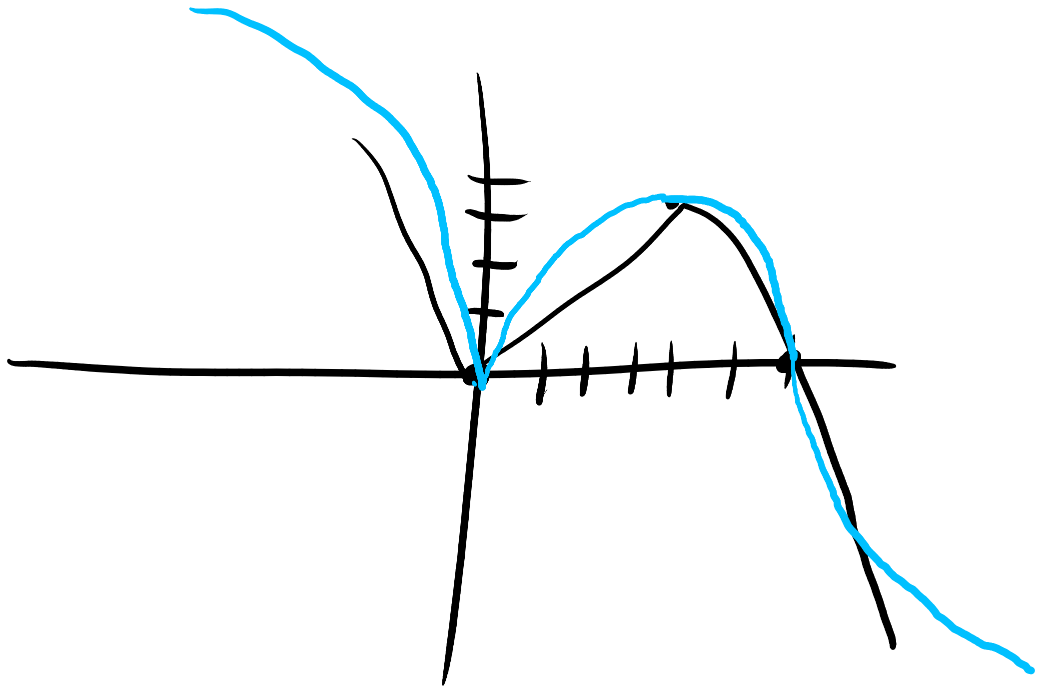
max at 4.

	$4-x$	$x^{1/3}$	$(6-x)^{2/3}$	$f'(x)$
$x<0$	+	−	+	−
$0<x<4$	+	+	+	+
$4<x<6$	−	+	+	−
$6<x$	−	+	+	−

min at 0

max at 4

neither at 6



	-8	$x^{4/3}$	$(6-x)^{5/3}$	f''
$x<0$	−	+	+	−
$0<x<6$	−	+	+	−
$6<x$	−	+	−	+

Curve Sketching

1. Find the domain of the function. If it has holes, what happens near them? Does it go to infinity, or jump, or just skip a point?

2. Find the roots–where does the function hit the x-axis?

3. Find the limits as x goes to $\pm\infty$–what happens to the function "far away" from 0?

4. Compute f' and find the critical points. It can be helpful to evaluate f at the critical points.

5. Find intervals of increase or decrease. Identify local maxima and minima.

6. Compute f'' if you haven't already. Determine where the function is concave, and find inflection points.

7. Use all this information to sketch a graph of the function.

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7. Use all this information to sketch a graph of the function.

$$f(x) = x^4 - 12x^3 + 48x^2 - 64x$$
$$= x(x-4)^3$$

$$f'(x) = (x-4)^2(4x-4)$$

$$f''(x) = (x-4)(12x-24) = 12(x-4)(x-2)$$

1) domain: all reals

2) roots: 0, 4

3) $\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = +\infty$

4) CP: 1, 4

$f(1) = -27$

$f(4) = 0$

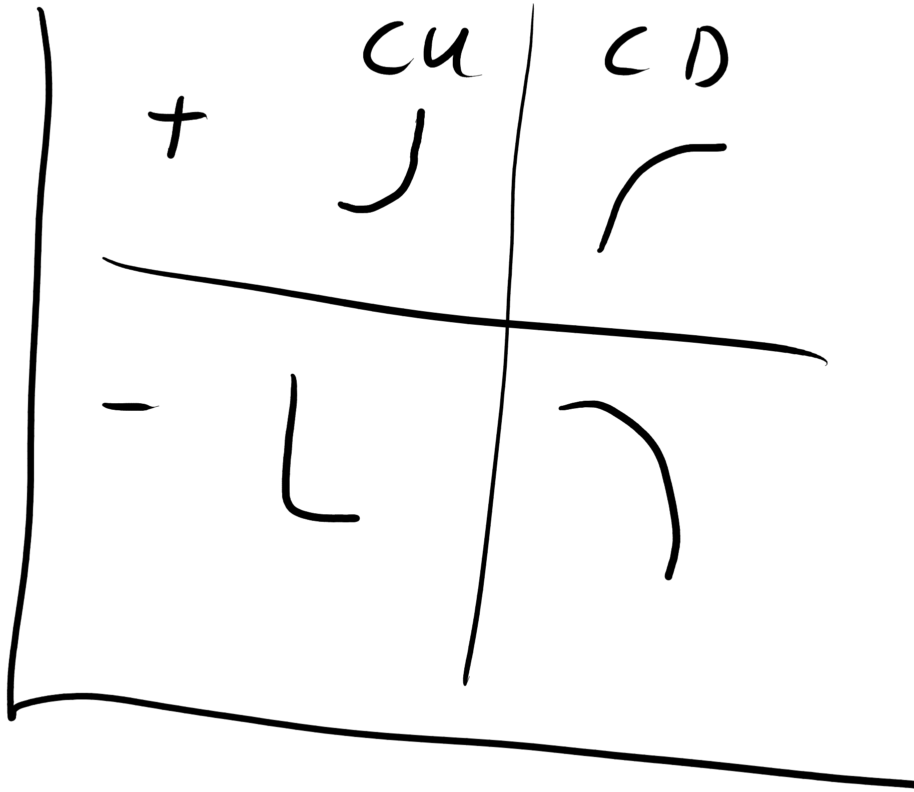
5)

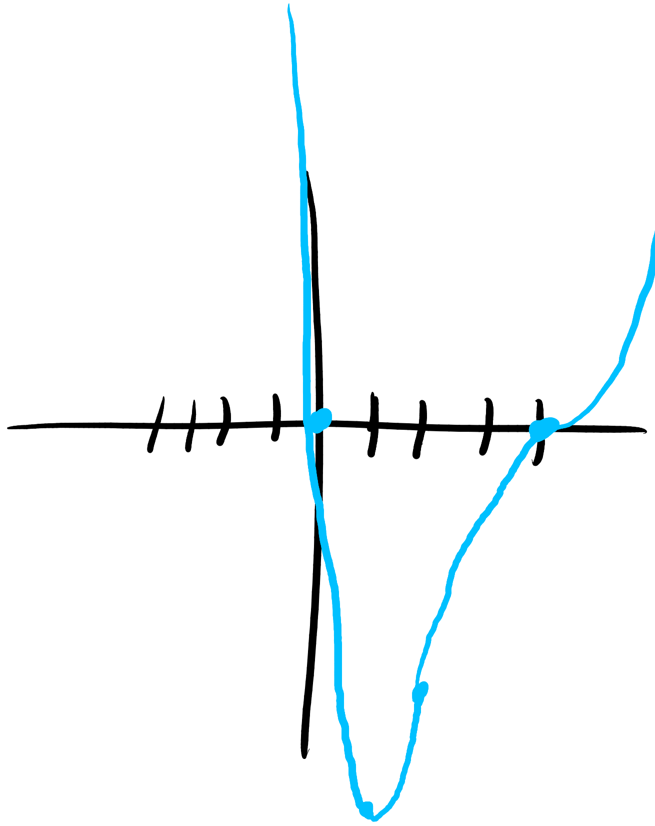
	$(x-4)^2$	$4x-4$	f'
$x < 1$	+	−	−
$1 < x < 4$	+	+	+
$4 < x$	+	+	+

6) PPOI: 2, 4; $f(2) = -16$, $f(4) = 0$

	$12(x-4)$	$x-2$	f''
$x < 2$	−	−	+
$2 < x < 4$	−	+	−
$4 < x$	+	+	+

POI: 2, 4





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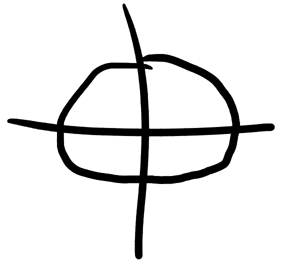
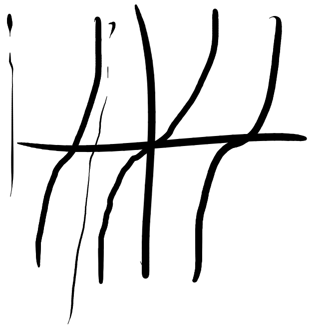
$g(x) = x\tan(x)$ on $(-3\pi/2, 3\pi/2)$

$g'(x) = \dfrac{\sin(x)\cos(x) + x}{\cos^2(x)}$ | $g''(x) = 2\sec^2(x)(1 + x\tan(x))$





1) $(-3\pi/2, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, 3\pi/2)$

$\lim_{x\to\pi/2^-} x\tan(x) = +\infty$

$\lim_{x\to\pi/2^+} x\tan(x) = -\infty$

$\lim_{x\to-\pi/2^-} x\tan(x) = \lim_{x\to-\pi/2^-} \dfrac{x\sin(x)}{\cos(x)} = -\infty$

$\lim_{x\to-\pi/2^+} x\tan(x) = +\infty$

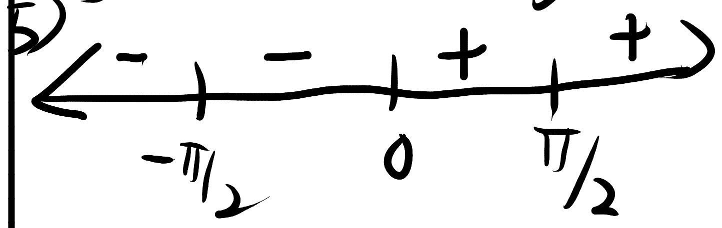
2) roots: $0, -\pi, \pi$

3) $\lim_{x\to 3\pi/2^-} x\tan(x) = +\infty$

$\lim_{x\to -3\pi/2^+} x\tan(x) = +\infty$

4) CP: $-\frac{\pi}{2}, \pi/2, 0$

5)



6) cu on $(-\pi/2, \pi/2)$