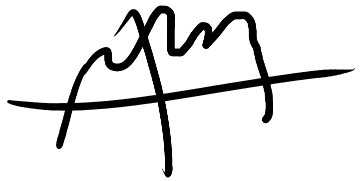
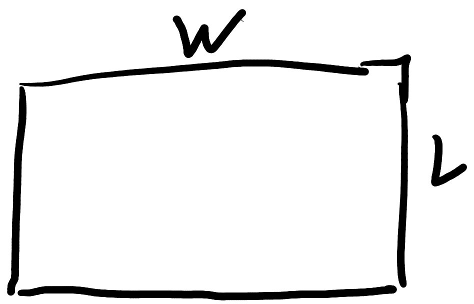
Optimization



Example 3.39. Suppose we have 2400 feet of fencing and we'd like to build a rectangular fence that encloses the most possible area. How can we do this?



$A = LW$

$2400 = 2L + 2W$

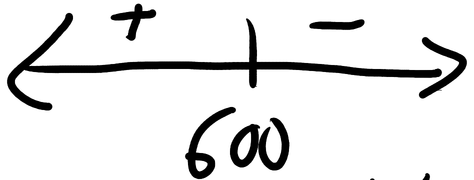
$L = 1200 - W$

$\Rightarrow A = W(1200 - W)$

$= 1200W - W^2$

$A' = 1200 - 2W$

CP: 600

1) 

global max b/c increasing all the way to 600 then ↓.

2) $A'' = -2 < 0$ always CD so only one max, global max.

3) A defined on $[0, 1200]$

By EVT, has global max

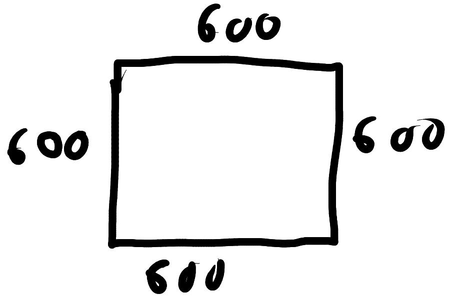
Check CP and endpts

$A(0) = 0$

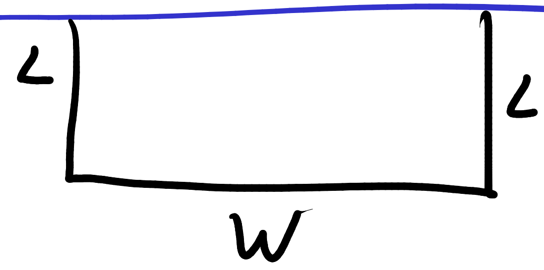
$A(1200) = 0$

$A(600) = 360{,}000$

So max at 600.



Instead:



$A = LW$

$2400 = 2L + W$

$W = 2400 - 2L$

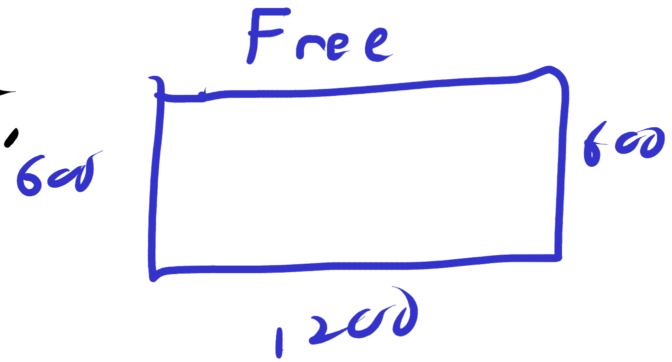
$A = L(2400 - 2L) = 2400L - 2L^2$

$A' = 2400 - 4L$

CP: $L = 600$

$W = 1200$

$A = 720{,}000 \text{ ft}^2$,



Example 3.40. Suppose we want to construct a cylindrical can that holds one liter of liquid, and we want to use the least possible metal to construct the can—and thus build the can with the least possible surface area. We have $A = 2\pi r^2 + 2\pi rh$.

obj fn: $A = 2\pi r^2 + 2\pi r h$

Constraint: $V = \pi r^2 h = 1$

$\Rightarrow h = \dfrac{1}{\pi r^2}$

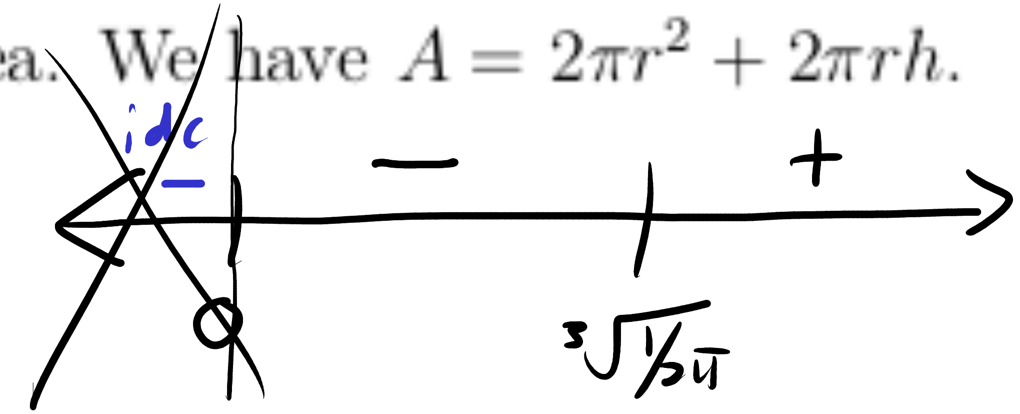
$A = 2\pi r^2 + \dfrac{2\pi r}{\pi r^2}$

$= 2\pi r^2 + 2/r$

$A' = 4\pi r - 2/r^2$

CP: $0, \sqrt[3]{1/2\pi}$

$4\pi r^3 = 2 \Rightarrow r = \sqrt[3]{1/2\pi}$



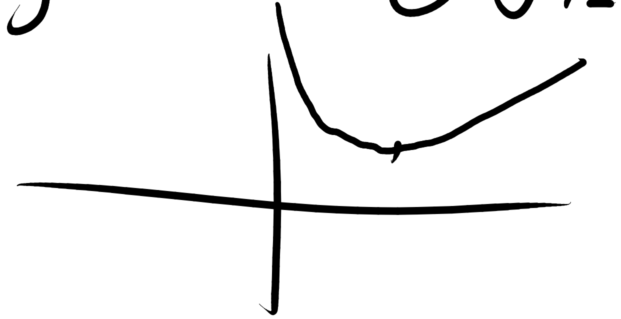
A defined on $(0, +\infty)$

Can't use EVT

decreasing to $\sqrt[3]{1/2\pi}$,

increasing after

so global min @ $\sqrt[3]{1/2\pi}$



What if sides cost twice as much as top/bottom?

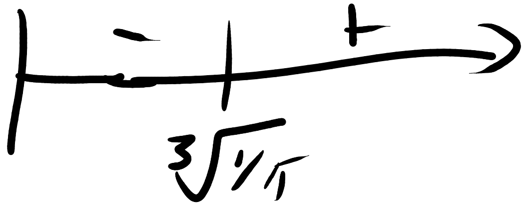
$C = 2\pi r^2 + 4\pi r h$

$h = 1/\pi r^2$

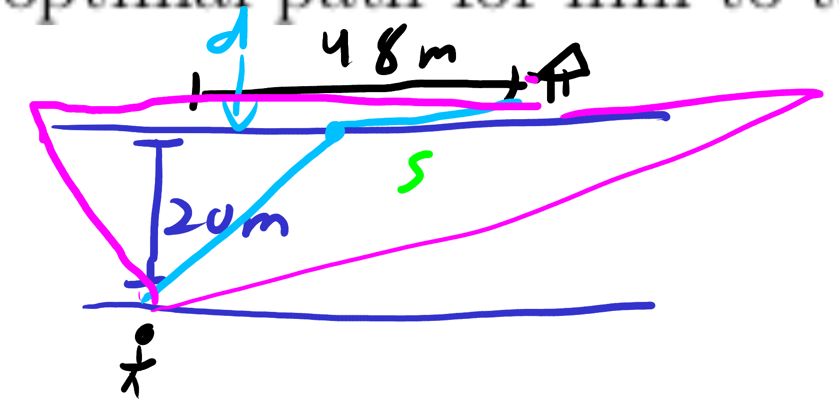
$C = 2\pi r^2 + 4/r$

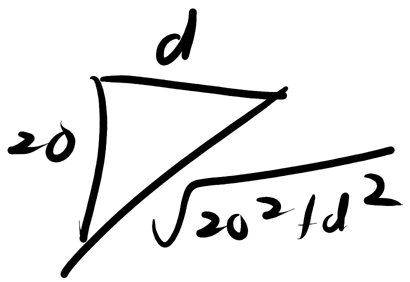
$C' = 4\pi r - 4/r^2$

CP: $0, \sqrt[3]{1/\pi}$



Example 3.42. Suppose a man wishes to cross a 20 m river and reach a house on the other side that is 48m downstream. The man can walk at 5 m/s or swim at 3 m/s. What is the optimal path for him to take to reach the house?





$$T = \frac{\text{walking}}{5} + \frac{\text{swim distance}}{3}$$

$$= \frac{48-d}{5} + \frac{\sqrt{20^2+d^2}}{3}$$

$$T' = \frac{-1}{5} + \frac{2d}{6\sqrt{400+d^2}}$$

$$\frac{2d}{6\sqrt{400+d^2}} = \frac{1}{5}$$

$$10d = 6\sqrt{400+d^2}$$

$$100d^2 = 36(400+d^2)$$

$$64d^2 = 36 \cdot 400$$

$$d^2 = 225$$

$$d = \pm 15$$

Actually just 15

have global min at 15

or: T defined on $[0,48]$

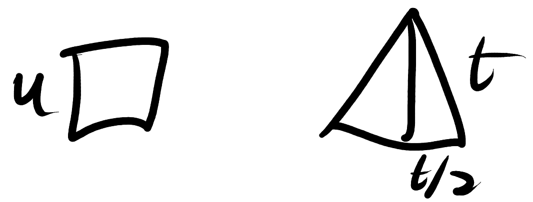
$$T(0) = \frac{48}{5} + \frac{20}{3} \approx 16.3$$

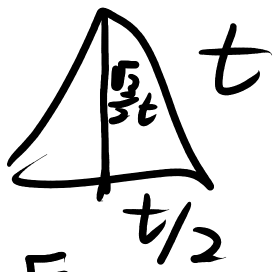
$$T(48) = \frac{\sqrt{20^2+48^2}}{3} = \frac{52}{3} \approx 17.3$$

$$T(15) \approx 14.9$$

$$\frac{5}{5} + \frac{\sqrt{20^2+(48-5)^2}}{3}$$

Example 3.43. A piece of wire 10 m long is going to be cut into two pieces. WE will fold one piece into a square and the other into an equilateral triangle. What is the largest joint area we can enclose? What is the smallest?





$A = u^2 + \frac{\sqrt{3}}{4} t^2$

$\frac{\sqrt{3}}{2} t (t) \frac{1}{2}$

$4u + 3t = 10$

$A = \left(\frac{10-3t}{4}\right)^2 + \frac{\sqrt{3}}{4} t^2$

$A' = 2\left(\frac{10-3t}{4}\right)(-3) + 2t\frac{\sqrt{3}}{4}$

$-15 + \frac{9t}{2} + \frac{\sqrt{3}\,t}{2} = 0$

$(9+\sqrt{3})t = 30$

$t = \frac{30}{9+\sqrt{3}}$

$A(cp) \approx 2.7$

$A(0) = (2.5)^2 = 6.25$

$A(\frac{10}{3}) \approx 4.8$

A defined on $[0, 10/3]$

min at $\frac{30}{9+\sqrt{3}}$

max @ 0