

Q: What is  $\sqrt{4}$ ? 2

Q: What is  $\sqrt{5}$ ? Uh...  
about 2

Fns like this are cts.

if  $a$  close to  $b$   
then  $f(a)$  close to  $f(b)$

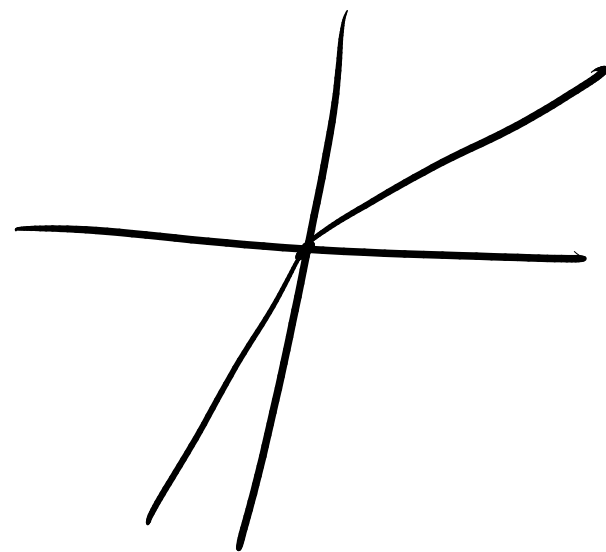
What is  $(3.125)^3 \approx 27$

real A: 30.52...

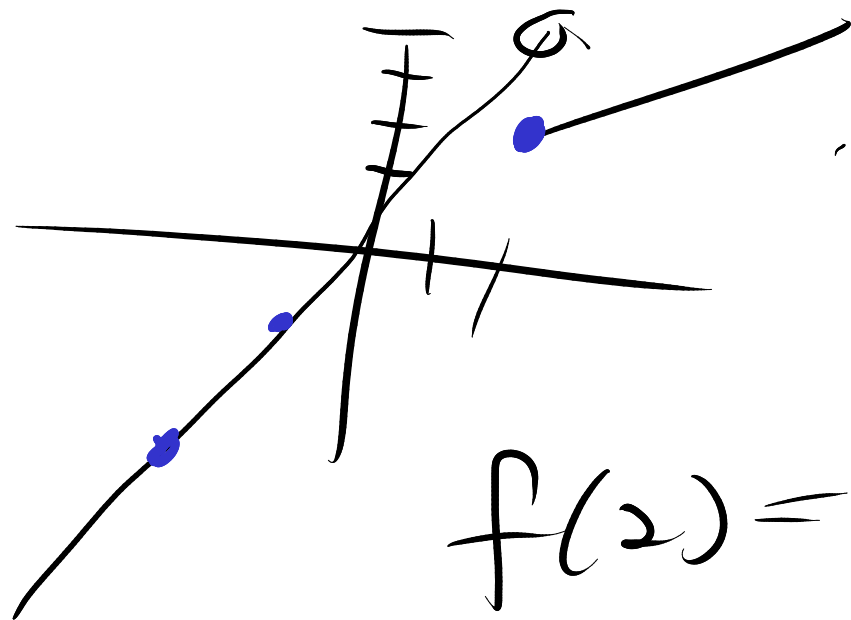
What is  $\sqrt[3]{28} \approx 3$

real  $\neq$  3.0365...

$$f_1(x) = \begin{cases} x & x \geq 0 \\ 2x & x < 0 \end{cases}$$



$$f_2(x) = \begin{cases} x & x \geq 2 \\ 2x & x < 2 \end{cases}$$

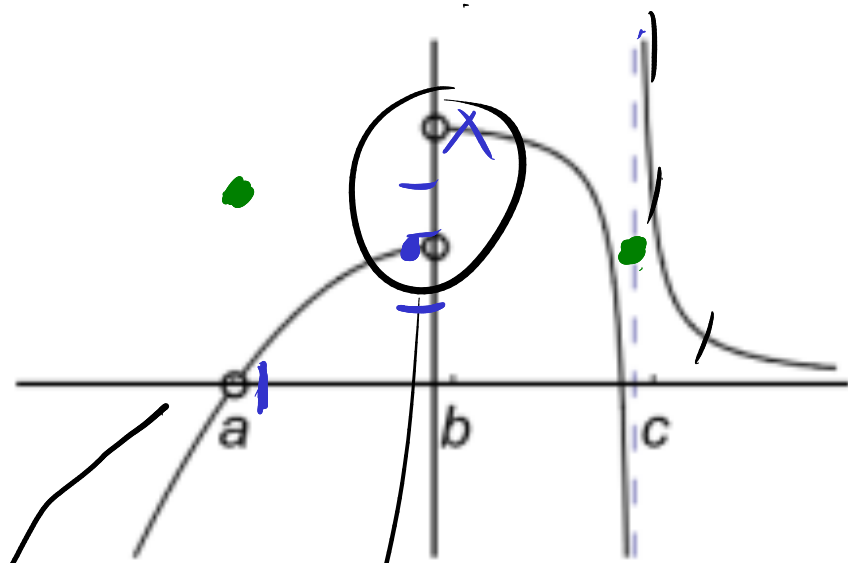


$$f(2) = 2$$

$$f(1.9) = 3.8 \neq 2$$

$$f(1.9999) = 3.9998 \neq 2$$

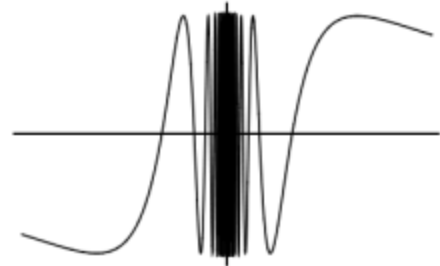
different things that can happen.



hole

jump

removable  
discontinuity



Can't say  $f(c) = \infty$  because  $\infty$  not a number

$$f(c + 0.0001) \approx \infty$$

$$= 9$$

$$= 999 \quad ??$$

$$= 999 \quad ??$$

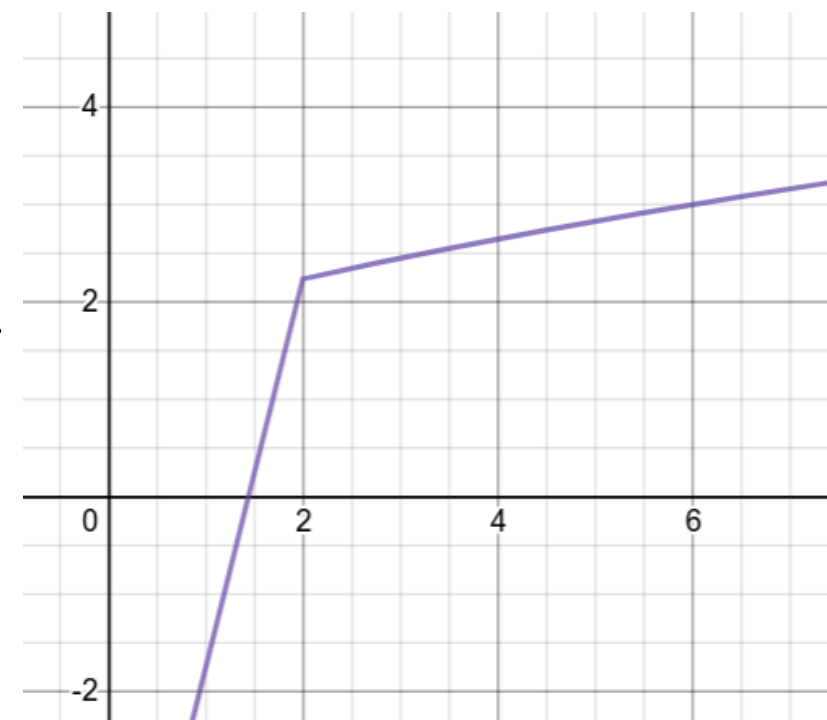
$f(c)$  DNE  
 $f(c + 0.0001)$  exists

$c$  not in  
 $\text{dom}(f)$

$$1) g(x) = \begin{cases} \sqrt{x+3} & x \geq 2 \\ 4x+b & x \leq 2 \end{cases} \quad \text{want } b = -6$$

for what  $b$  is this cts?

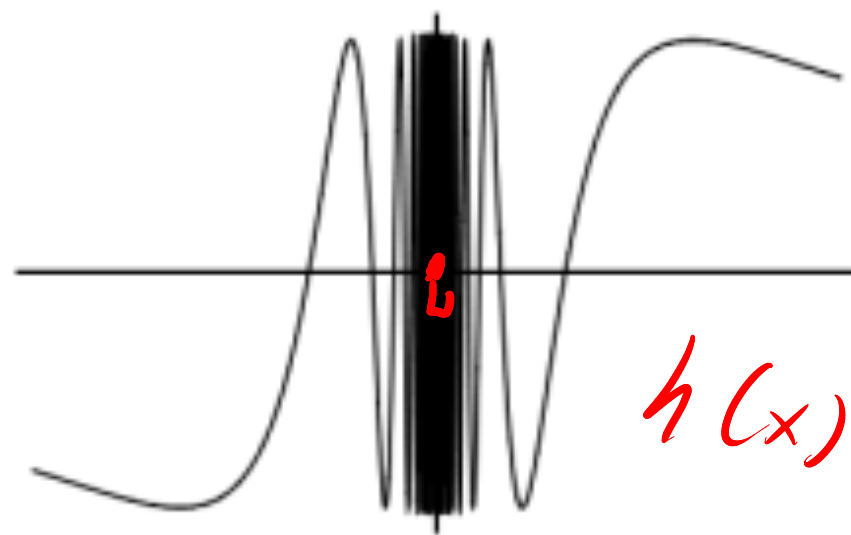
$$g(2) = \sqrt{5} \quad b = \sqrt{5} - 8$$



$$2) h(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ c & x = 0 \end{cases}$$

Can I pick  $c$

so  $h$  is cts



in  $[-.0001, .0001]$

in  $[-.00000001, .00000001]$

$h(x)$  in  $[-.1, .1]$

# limits

$$\text{Dfn. } \lim_{x \rightarrow a} f(x) = L$$

$$x \rightarrow a$$

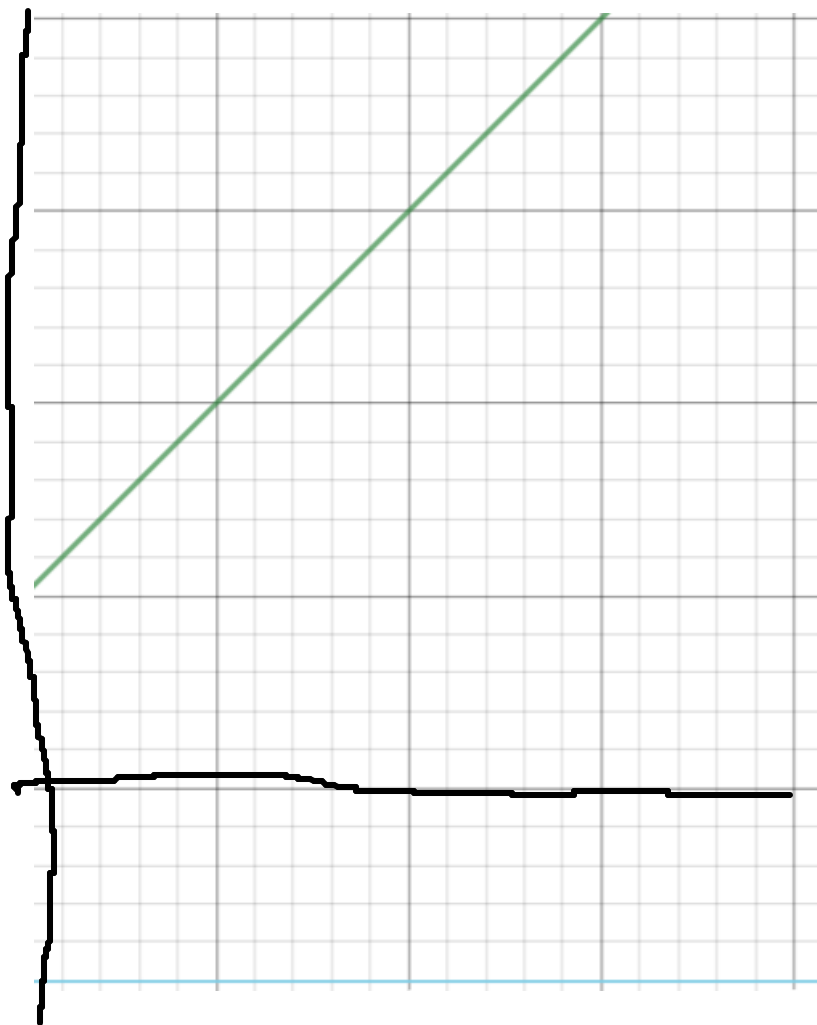
if

$f(x)$  gets as close to  $L$   
as we want  
when  $x$  is close enough  
to  $a$ .

$$\lim_{x \rightarrow 1} x+1$$

b/c cts,

$$\lim_{x \rightarrow 1} f(x) = f(1) = 2$$



$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

~~$= 1$~~

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0} = 1$$

*(The above equation is crossed out with red lines)*

$$f \neq g$$

$$\text{DNE} = f(1) \neq g(1) = 2$$

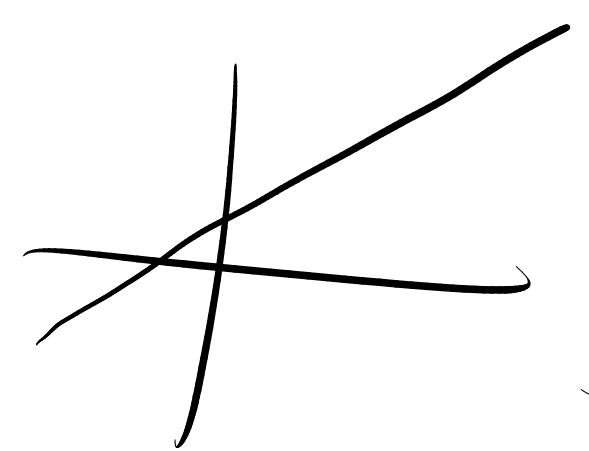
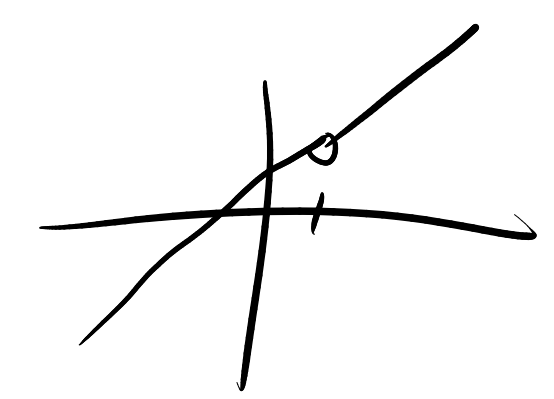
$$\lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1} x+1$$

$f(x) = g(x)$   
for  $x$  near 1

$$f(1) = \frac{0}{0} \text{!!!}$$

$$g(1) = 2$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} g(x) = 2$$



what should happen  
at 1

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \quad \frac{f(1) - f(0)}{1} \quad \begin{array}{l} \leftarrow \text{miles} \\ \leftarrow \text{hours} \end{array}$$

$$\text{speed 'at' noon?} \quad \frac{f(0) - f(0)}{0 - 0}$$

$$\frac{f(.001) - f(0)}{.001 - 0}$$

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$$