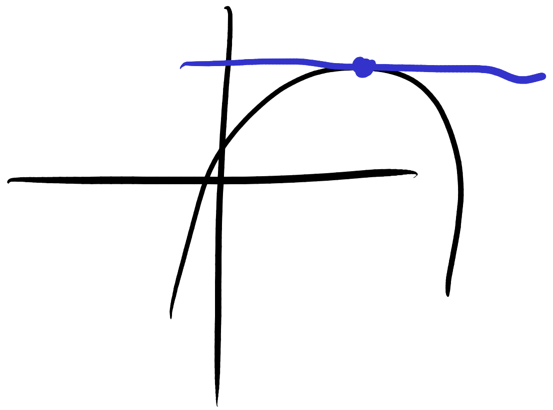
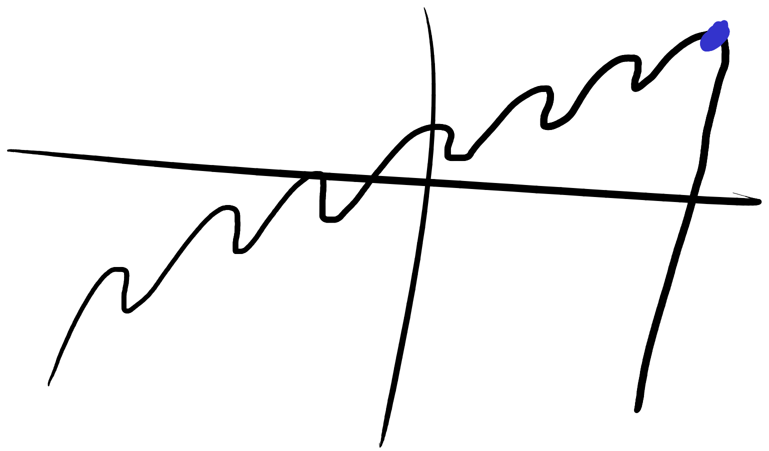
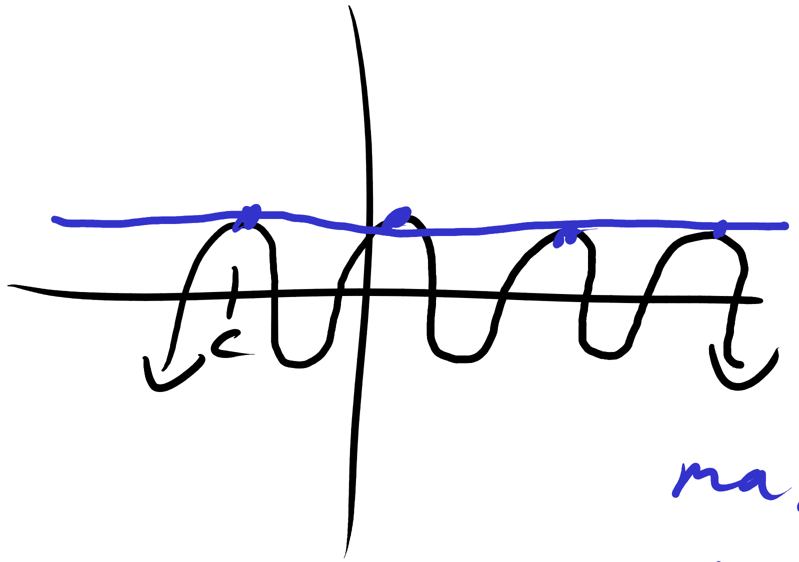
§3 Optimization

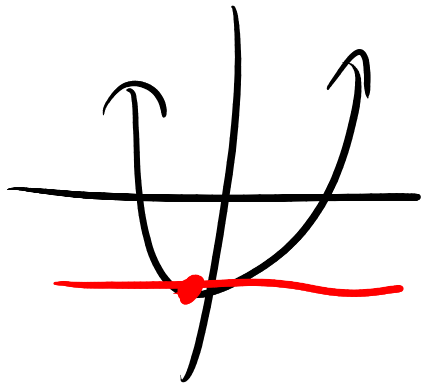


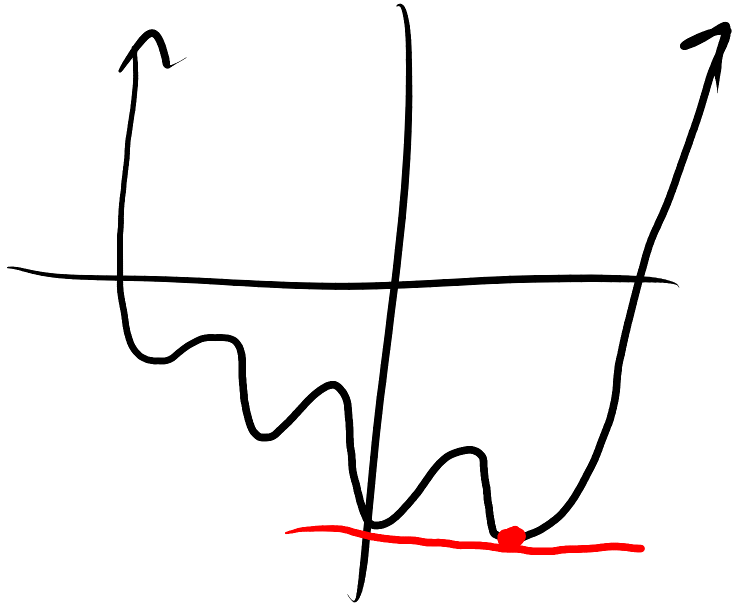


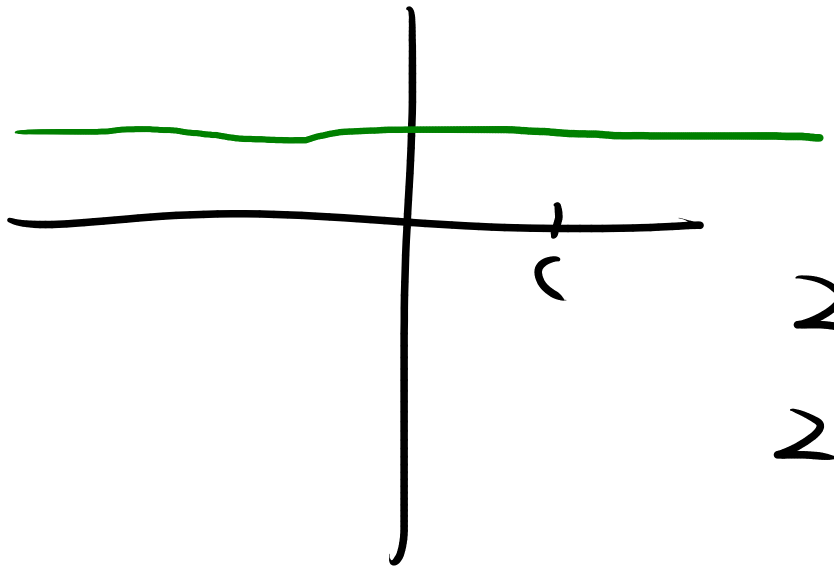


one max
multiple ways
to achieve

maxes are
outputs/y-values







$y = 2$

2 is the max

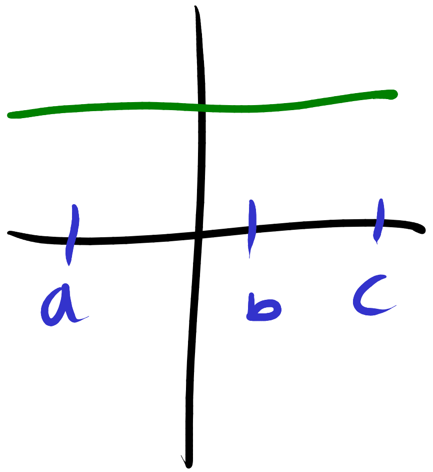
2 is the min

Dfn: If $f(c) \geq f(x)$ for every x

say $f(c)$ is an absolute max or global maximum.

If $f(c) \leq f(x)$ for every x

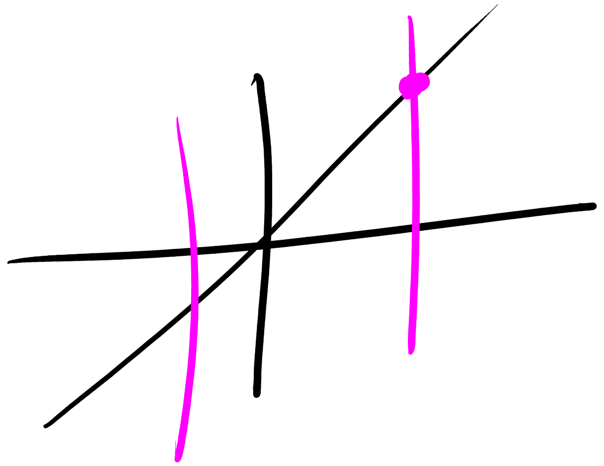
say $f(c)$ is an absolute min or global min.



$2 = f(a)$ is global max

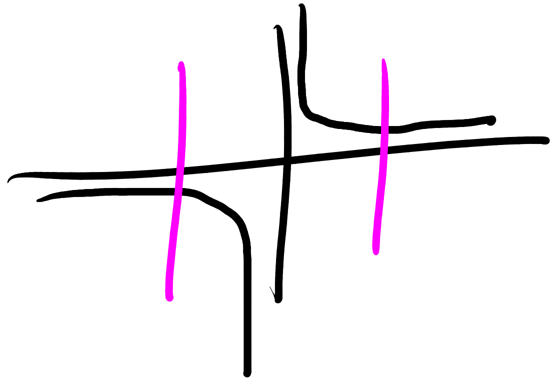
$2 = f(b)$ is a global max.

$2 = f(c)$ is a global max



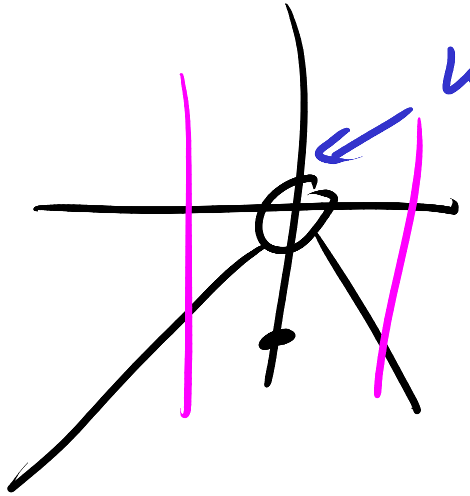
$f(x) = x$

no max



$1/x$

no max

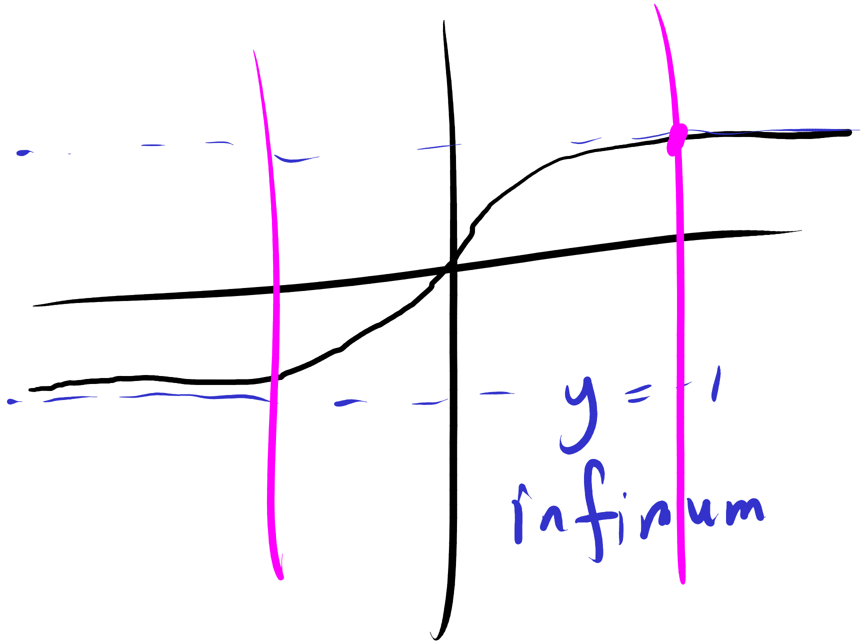


wants to be max

supremum

$-|x|$

no maximum



$$\frac{x}{\sqrt{x^2+1}}$$

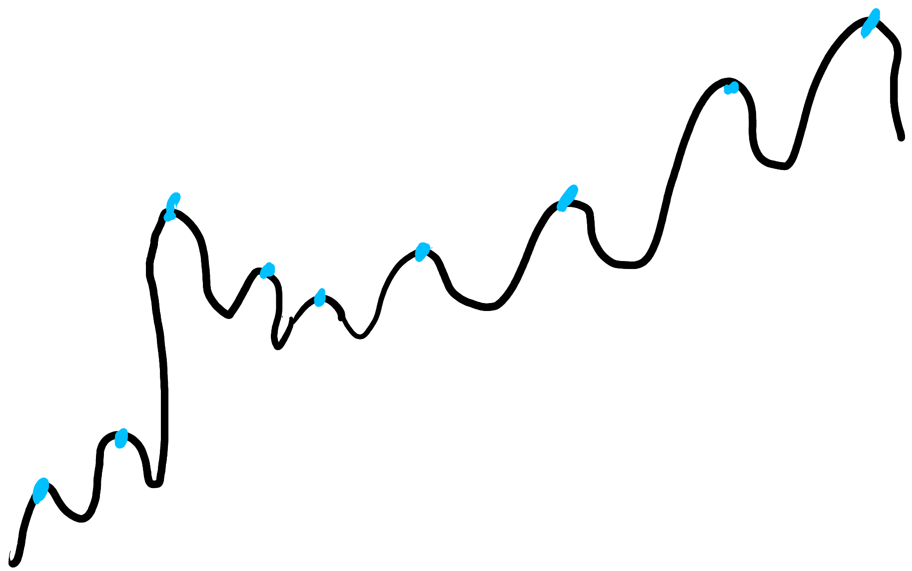
no max

no min.

Extreme Value Thm

If f is cts on $[a,b]$, then f has abs max $f(c)$ for some c in $[a,b]$ and an abs min $f(d)$ for some d in $[a,b]$

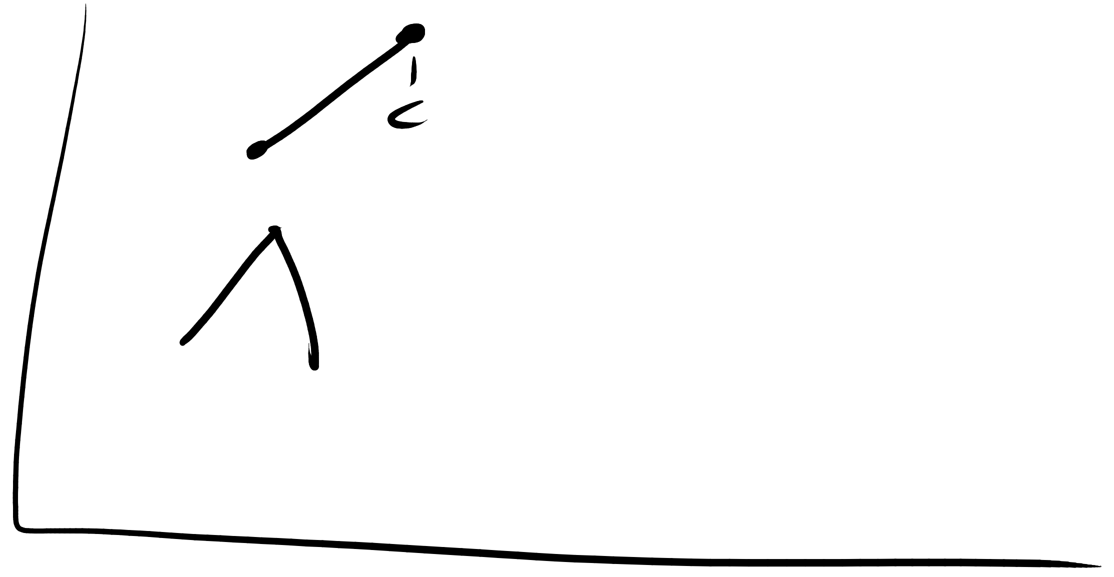
Dfn: If $f(c) \geq f(x)$ for x near c,
then $f(c)$ is a relative or local max
and f has a (local) max at c.



Every global max
is a local max

Thm (Fermat's Thm / Critical pt thm):

If f has a local extremum at c,
and c is not an endpt
and $f'(c)$ exists,
then $f'(c)=0$.



"Pf" Suppose c has a local max
x is near c. Then $f(x) \leq f(c)$

$f(x) \approx f(c) + f'(c)(x-c)$.

If $f'(c) > 0$, $f(c+\delta) > f(c)$, not local max

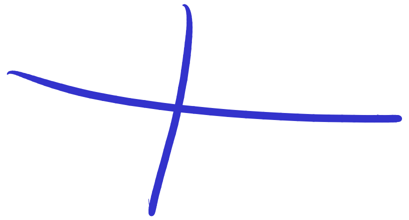
If $f'(c) < 0$, $f(c-\delta) > f(c)$, not a local max

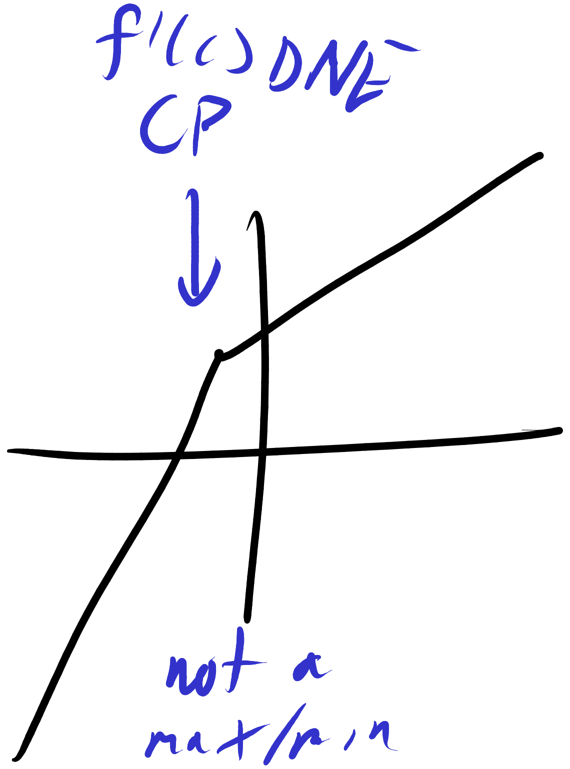
so if $f'(c)$ exists, then $f'(c) = 0$

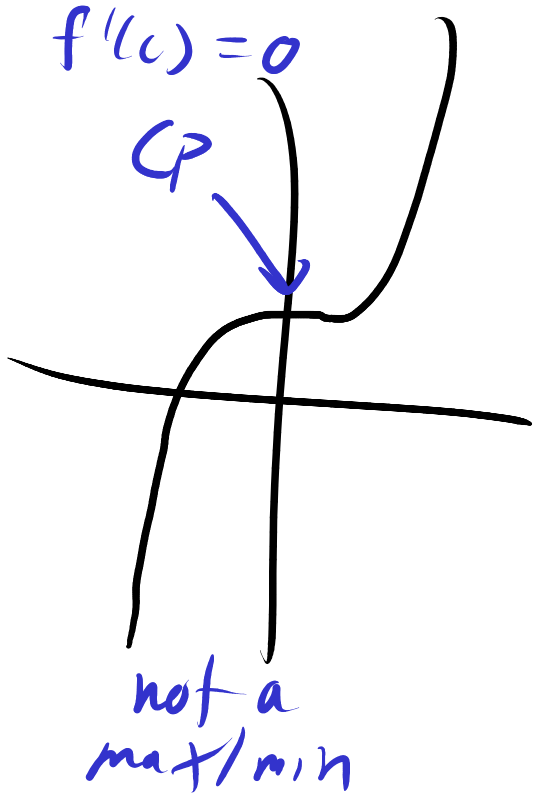
Dfn: c is a <u>critical pt</u> of f

if either • $f'(c) = 0$

or • $f'(c)$ DNE.







every max/min is a CP.

(or endpt)

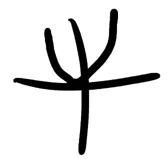
$f(x) = x^2$

DNE: no

$f'(x) = 2x$

$f' = 0$: $x = 0$

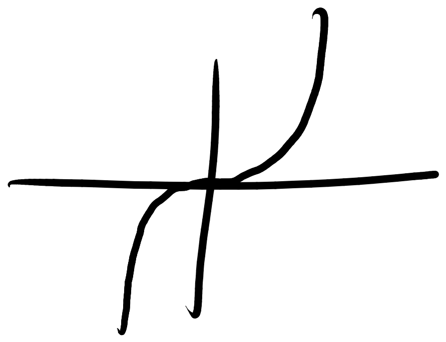
CP: 0.



$g(x) = x^3$

$g'(x) = 3x^2$

CP: 0



$h(x) = \sin(x)$

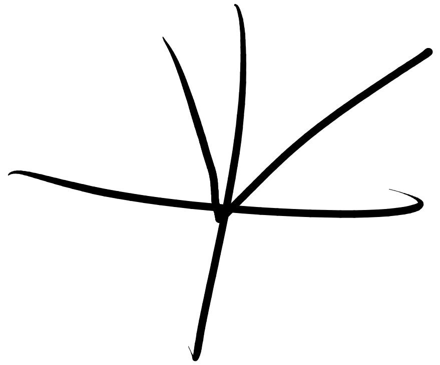
$h'(x) = \cos(x)$

CP: $\pi/2$, $3\pi/2$, $5\pi/2$, $-\pi/2$, etc

CP: $n\pi + \pi/2$

$f(x) = |x|$

$$f'(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \\ \text{DNE} & x = 0 \end{cases}$$



CP: 0

$g(x) = \sqrt[3]{x}$

$g'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

CP: 0

How do I find global maxima
1) find CP and endpts
2) which is biggest?

Let $f(x) = x^3 - x$ on $[0, 2]$.

f is cts on closed interval so it <u>has</u> max/min by EVT.

$f'(x) = 3x^2 - 1$
exists everywhere

Solve $3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm\sqrt{1/3}$

$CP = \pm\sqrt{1/3}$

only care about $+\sqrt{1/3}$

$f(0) = 0$

$f(\sqrt{1/3}) = \frac{-2}{3\sqrt{3}}$

$f(2) = 6$

max: 6 @ 2

min: $\frac{-2}{3\sqrt{3}}$ @ $\sqrt{1/3}$