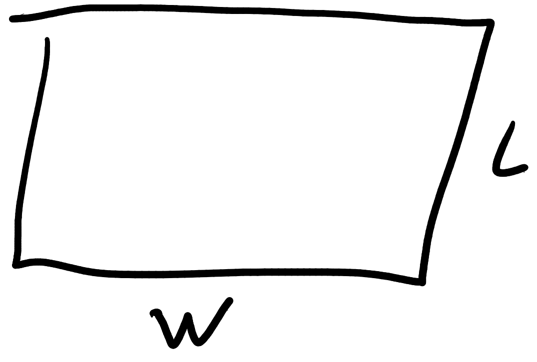
**Example 3.39.** Suppose we have 2400 feet of fencing and we'd like to build a rectangular fence that encloses the most possible area. How can we do this?




$A = L \cdot W$

$2L + 2W = 2400$

$W = 1200 - L$

Maximize

$A = L(1200 - L)$

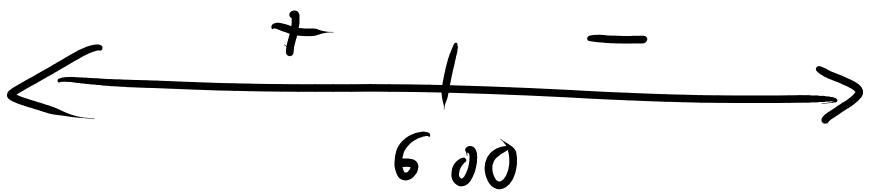
$= 1200L - L^2$

$A' = 1200 - 2L$

CP: $L = 600$.

1) $A' > 0$ if $L < 600$

$A' < 0$ if $L > 600$




600 must be a max

decreasing in either direction.

2) $A'' = -2 < 0$

A is concave down

- def rel max

since CD everywhere, global max?

3) A is defined on $[0, 1200]$.

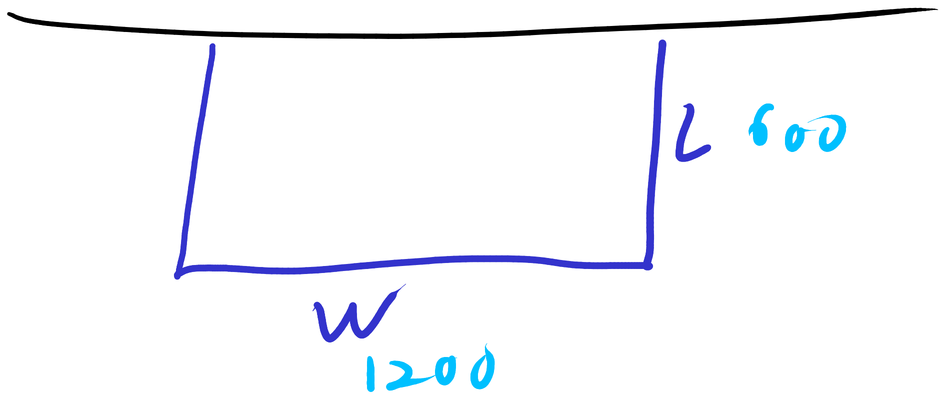
by EVT, has a max.

$A(600) = 360{,}000$

$A(0) = 0 \qquad A(1200) = 0$

So 360,000 is largest value.

what if we can build against a cliff?




$A = L \cdot W$

$2400 = 2L + W$

$W = 2400 - 2L$

$A = L(2400 - 2L)$

$= 2400L - 2L^2$

$A' = 2400 - 4L$

CP: $L = 600$

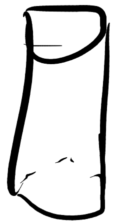
A is defined on $[0, 1200]$

$A(0) = 0$, $A(1200) = 0$

$A(600) = 720{,}000$.

is the max.

**Example 3.40.** Suppose we want to construct a cylindrical can that holds one liter of liquid, and we want to use the least possible metal to construct the can—and thus build the can with the least possible surface area. We have $A = 2\pi r^2 + 2\pi r h$.



minimize $A = 2\pi r^2 + 2\pi r h$ Objective Function

$\pi r^2 h = 1$. Constraint eqn

$h = \frac{1}{\pi r^2}$

$\Rightarrow A = 2\pi r^2 + \frac{\cancel{2\pi r}}{\cancel{\pi r^2}}$ $\frac{2}{r}$

$A' = 4\pi r - \frac{2}{r^2}$

CP: $\cancel{0}$, $\sqrt[3]{1/2\pi}$

$4\pi r = \frac{2}{r^2}$

$4\pi r^3 = 2$

$r = \sqrt[3]{\frac{1}{2\pi}}$

A defined on $(0, +\infty)$

EVT doesn't work

$-$ $+$

$\sqrt[3]{1/2\pi}$

$A'(1/1000) < 0$

$A'(1000) > 0$

So we have a (global) min at $\sqrt[3]{1/2\pi}$

So imagine side area costs twice as much.

min $C = 2\pi r^2 + 4\pi r h$

$C = 2\pi r^2 + \frac{4}{r}$

$C' = 4\pi r - \frac{4}{r^2}$

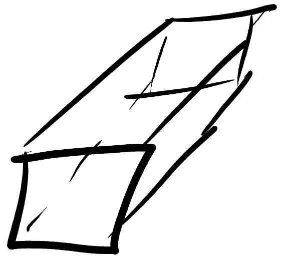
$4 = 4\pi r^3$

$\Rightarrow r = \sqrt[3]{1/\pi}$.

$-$ $+$

global min at $\sqrt[3]{1/\pi}$.

**Example 3.41.** If we have 1200 $\text{cm}^2$ of cardboard to make a box with a square base and an open top, what is the largest possible volume of the box?



Obj: $V = L \cdot W \cdot H$

$L = W$

$1200 = L^2 + 4 L \cdot H$

$\Rightarrow H = \dfrac{1200 - L^2}{4L}$

$V = L^2 \dfrac{1200 - L^2}{4L} = 300 L - L^3/4$

$V' = 300 - \dfrac{3L^2}{4}$

$1200 = 3L^2$

$400 = L^2$

$L = \pm 20$

$V$ defined on $[0, \sqrt{1200}]$

by EVT, has a max

$V(0) = 0$

$V(\sqrt{1200}) = 0$

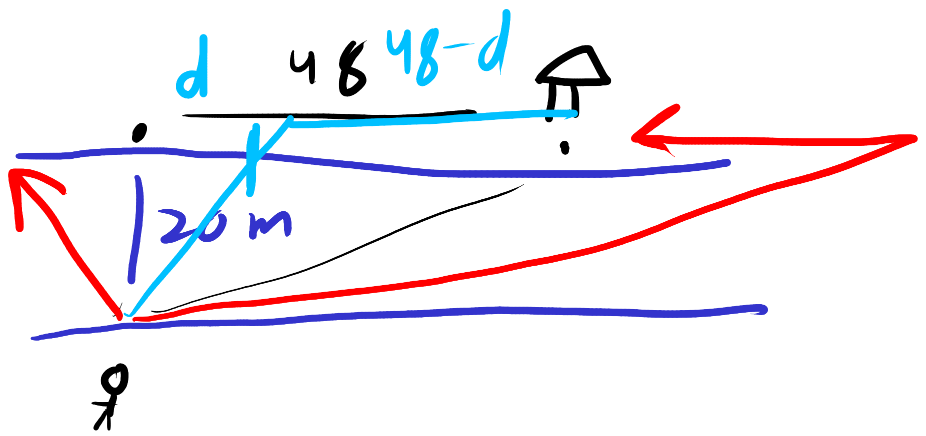
$V(20) = 6000 - 2000$

$= 4000$

max Vol

$L = 20$, $H = 10$.

**Example 3.42.** Suppose a man wishes to cross a 20 m river and reach a house on the other side that is 48m downstream. The man can walk at 5 m/s or swim at 3 m/s. What is the optimal path for him to take to reach the house?




$$T = \frac{\text{swim distance}}{3} + \frac{\text{walk distance}}{5}$$

$$WD = 48 - d$$

$$SD = \sqrt{20^2 + d^2}$$

$$T = \frac{\sqrt{400 + d^2}}{3} + \frac{48 - d}{5}$$

$$T' = \frac{2d}{6\sqrt{400+d^2}} + \frac{-1}{5}$$

$$= \frac{d}{3\sqrt{400+d^2}} - \frac{1}{5}$$

$$5d = 3\sqrt{400 + d^2}$$

$$25d^2 = 3600 + 9d^2$$

$$16d^2 = 3600$$

$$d^2 = 225$$

$$d = \pm 15$$

$-$ $-$ $+$

$-15$ $15$

$d(0) = -1/5 < 0$

$d(\text{house}) \approx 4/5 > 0$

reasonable to define $T$ on $[0, 48]$

$$T(0) = \frac{20}{3} + \frac{48}{5} \approx 16.3$$

$$T(48) = \frac{\sqrt{20^2+48^2}}{3} = \frac{52}{3} \approx 17.3$$

$$T(15) = \frac{\sqrt{20^2+15^2}}{3} + \frac{33}{5} \approx 14.9$$

min time is $\approx 14.9$ s