Math 1231 Final

Instructor: Jay Daigle

December 15-16, 2020

1. This test is due at the scheduled exam time. Logistically, this will work just like the mastery quizzes: download it, write up your answers, and upload them to Blackboard for us to grade.
2. You will have two hours for this test. Please write down your start and end times on the test and include that in your upload. You may not spend more than two hours on the test unless you have a specific accommodation.
3. You are not allowed to consult books or notes during the test, but you may use a one-page cheat sheet you have made for yourself ahead of time. Please upload your sheet along with your test.
4. If you have questions, I will be online and responsive during the scheduled exam time. If you want to take the test at a time you know I'll be able to answer any questions quickly, I encourage you to use that time slot.
5. You may use a calculator, but don't use a graphing calculator or anything else that can do symbolic computations. Using a calculator for basic arithmetic is fine.

Name:

Time Started:

Time Completed:

Problem 1. (a) A curve is defined by the equation $2\sin(y) - xy + y^2 = 0$. Verify that the curve passes through the point (π, π). What is the equation of the tangent line to the curve at this point?

(b) Find all the critical points of $g(x) = \sqrt[3]{x^3 - 3x^2 + 2x}$.

(c) Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-3, 3]$.

Problem 2. (a) Classify the relative extrema of $\dfrac{x^2}{x-2}$.

(b) A boat leaves a dock at 2 PM and travels due south at a speed of 20km/h. Another boat has been heading due east at 20km/h, and reaches the same dock at 3 PM. At what time were the two boats closest together? **Justify your claim** that this is a minimum.

Problem 3. (a) If $f(x) = \sqrt{x^2 + 1}$, use a quadratic approximation centered at 0 to estimate $f(.2)$.

(b) Use two iterations of Newton's Method to estimate a solution to $x^3 + 3x + 3 = 0$, starting with $x_0 = 0$.

(c) Using three rectangles and right endpoints, approximate the area under the curve $\frac{x}{x+1}$ between $x = 2$ and $x = 5$.

Problem 4. (a) Using **only the definition of Riemann sum** and your knowledge of limits, compute the exact area under the curve $2x^2$ between $x = -2$ and $x = 1$.

(b) Let $G(x) = \int_{-3}^{\sin(x^2)} \sqrt{t + t^2}\, dt$. What is $G'(x)$?

(c) Compute $\displaystyle\int \left(x - \frac{1}{x^2}\right)\left(x^2 + \frac{2}{x}\right)^4 dx$

Problem 5. (a) **By explicitly changing the bounds of the integral**, compute $\int_1^2 x\sqrt{x-1}\,dx$.

(b) Sketch the region bounded by the curves $y = (x-1)^2$ and $y = x+1$. Find the area of the region.

(c) What is the average value of the function $h(x) = x + \sqrt{x}$ on the interval $[1, 4]$?