Problem 1. Compute the following limits if they exist. Show enough work to justify your computation, or your claim that the limit does not exist.

(a)

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x}$$

(b)

$$\lim_{x\to -\infty} \frac{3x^3+\sqrt[3]{x}}{\sqrt{9x^6+2x^2+1}+x}$$

(c)

$$\lim_{x\to 1} \frac{\sin^2(x-1)}{(x-1)^2} =$$

(d)

$$\lim_{x\to 3} \frac{x-5}{(x-3)^2} =$$

Problem 2.

(a) **Directly from the definition of derivative,** compute the derivative of $f(x) = x^2 + \sqrt{x}$ at $a = 2$.

(b) Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

(i) What does the derivative $Q'(p)$ represent, and what are its units?

(ii) Calculate $Q'(10)$. What does this tell you?

Problem 3.

(a) Find an equation of the line tangent to $y = \frac{x^2-1}{x^2+1}$ at the point $(0, -1)$.

(b) Give equation for the linear approximation of the function $f(x) = x\sin(x)$ near the point $a = \pi/2$.

Problem 4. Compute the derivatives of the following functions using methods we have learned in class. Show enough work to justify your answers.

(a) $f(x) = \sec\left(\frac{\sqrt{x^2+1}}{x+2}\right)$

(b) $g(x) = \sqrt[4]{\frac{x^3+\cos(x^2)}{\sin(x^3)+1}}$

Problem 5. (a) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

(b) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?