Math 1231 Fall 2020
Single-Variable Calculus I Mastery Quiz 10
Due midnight on Thursday, November 12

This week's mastery quiz has twelve topics. (Or at least it will some time soon!) **Do not answer all ten.** You may answer the first question on the newest topic, numbered sixteen, and *two* additional topics. You may pick two topic you have not yet demonstrated mastery on and answer the questions on that topic. (If you are retrying a topic, please complete the entire page.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics:

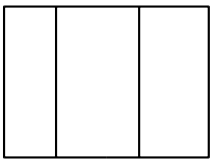
16. Optimization
15. Curve Sketching
14. First and Second Derivative Tests
13. Global Maxima and Critical Points
12. Related Rates
11. Implicit Differentiation
10. Rates of Change

To come:

6. Definition of a Derivative
5. Infinite Limits
4. Trigonometric Limits
3. Computing Limits
2. Formal limits

16. **Optimization**

We wish to build a rectangular pen with two parallel internal partitions, using 1000 feet of fencing. What dimensions maximize the total area of the pen?



15. **Curve Sketching**

Sketch the graph of $f(x) = x^5 - 5x^4 + 5x^3 = x^3(x^2 - 5x + 5)$. We have $f'(x) = 5x^2(x-3)(x-1)$ and $f''(x) = 10x(2x^2 - 6x + 3)$.

You should discuss the domain, limits, critical points, intervals of increase and decrease, concavity, and possible points of inflection.

14. **First and Second Derivative Tests**

(a) Classify all the critical points and relative extrema of $h(x) = x^3/(x+1)$. (For each critical point, tell me whether it is a relative maximum, a relative minimum, or neither.)

(b) Classify the critical points and relative extrema of $h(x) = \sin(x) + \cos(x)$ on $[0, 2\pi]$.

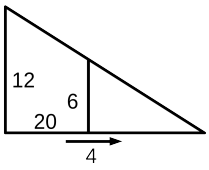
13. **Global Maxima and Critical Points**

(a) Find the absolute extrema of $f(x) = x^3 + x^2 - 5x$ on $[-1, 2]$.

(b) Find all the critical points of $g(x) = \sqrt[3]{x^2 - 3x + 2}$.

12. **Related Rates**

A street light is mounted at the top of a 12-foot-tall pole. A six-foot-tall man walks straight away from the pole at 4 feet per second. How fast is his shadow growing longer when he is twenty feet from the pole?



11. **Implicit Differentiation**

(a) Write a tangent line to the curve $x^2y^2 = 5 + x + y$ at the point $(1, 3)$.

(b) Find a formula for y' in terms of x and y if $xy^3 = \sqrt{x^2 + y^2}$.

10. **Rates of Change**

(a) Suppose that a factory produces widgets, and if p people work at the factory then they will produce a total of $W(p) = 30\sqrt{p}$ widgets.

(i) What does the derivative $W'(p)$ represent, and what are its units?

(ii) Calculate $W'(9)$. What does this represent in the real world?

(b) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula $d(t) = t^3 + 4t^2 + 5t + 4$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

6. **Definition of a Derivative**

Compute the following derivatives, *directly from the formal definition of derivative.*

(a) If $f(x) = \sqrt{2x+3}$, find $f'(3)$.

(b) If $g(x) = \frac{3}{x^2}$, find $g'(x)$.

5. **Infinite Limits** Compute:

(a) $\lim_{x \to +\infty} \dfrac{x(2x+3)(3x-2)}{x^3+x^2-3x+4} =$

(b) $\lim_{x \to 2^+} \dfrac{x+1}{x-2} =$

(c) $\lim_{x \to 1} \dfrac{x-5}{x-1} =$

4. **Trigonometric Limits**

(a) Show that $\lim_{x\to 1}(x-1)^2\left(1+\sin\left(\frac{2}{x-1}\right)\right)=0$.

(b) Compute $\lim_{x\to 3}\frac{\sin(x-3)\sin(6x-18)}{(x-3)^2}$.

3. **Computing Limits** Compute:

(a) $\lim_{x \to 7} \dfrac{\sqrt{9+x} - 4}{x - 7} =$

(b) $\lim_{x \to -2} \dfrac{1}{x+2} + \dfrac{1}{x^2 + 3x + 2}$

(c) $\lim_{x \to 1} \dfrac{x^2 - 5x + 4}{x^2 + 6x - 7} =$

2. **Formal Limits**

(a) Write a formal ϵ-δ proof that $\lim_{x\to 2} 2x + 2 = 4$.

(b) Explicitly naming each limit law you use, compute

$$\lim_{x\to 3} \frac{3x^2 + 5}{4x - 2} =$$