# Math 1231 Fall 2020
# Single-Variable Calculus I Mastery Quiz 11
# Due midnight on Thursday, November 19

This week's mastery quiz has seven topics. **Do not answer all five.** You may answer the two newest topics, numbered 18 and 17, and *one* additional topics. You may pick one topic you have not yet demonstrated mastery on and answer the questions on that topic. (If you are retrying a topic, please complete the entire page.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics:

18. Area and Riemann Sums
17. Approximation
16. Optimization
15. Curve Sketching
14. First and Second Derivative Tests
13. Global Maxima and Critical Points
9. Linear Approximations and Tangent Lines

18. **Area and Riemann Sums**

Let $f(x) = x^2 - x$ be defined on the interval $[-3, 0]$.

(a) Approximate the area under the curve of the function using three rectangles and right endpoints.

(b) Approximate the area under the curve of the function using three rectangles and left endpoints.

(c) Find a formula for computing $R_n$, the estimate using $n$ rectangles and right endpoints. (This formula should not have a summation sign or be given as a sum of $n$ terms.)

(d) Use the formula in part (c) to compute the area exactly.

17. **Approximation**

(a) Find a formula for the quadratic approximation of $f(x) = \sqrt{3x+1}$ near the point $a = 1$, and use it to estimate $f(1.01)$.

(b) Use two steps of Newton's method to estimate $\sqrt{8}$ starting from $x_0 = 3$. (You should compute $x_2$.)

16. **Optimization**

Suppose that a company that produces and sells $x$ units of a product makes a revenue of $R(x) = 260x - 9x^2/10$ and has costs given by $C(x) = 1000 + 100x + x^2/10$. What is the maximum profit that can be made (where profit is revenues minus costs)?

15. **Curve Sketching**

Sketch the graph of $g(x) = 3x^4 - 4x^3 - 36x^2 + 64 = (x+2)^2(3x-4)(x-4)$ have $g'(x) = 12x^3 - 12x^2 - 72x = 12x(x-3)(x+2)$ and $g''(x) = 36x^2 - 24x - 72 = 12(3x^2 - 2x - 6)$.

You should discuss the domain, limits, critical points, intervals of increase and decrease, concavity, and possible points of inflection.

14. **First and Second Derivative Tests**

(a) Classify all the critical points and relative extrema of $f(x) = \frac{x}{x^2+1}$. (For each critical point, tell me whether it is a relative maximum, a relative minimum, or neither.)

(b) Classify the critical points and relative extrema of $g(x) = \cos^2(x) - 2\sin(x)$ on $[0, 2\pi]$

13. **Global Maxima and Critical Points**

(a) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

(b) Find all the critical points of $g(x) = \dfrac{x^2 - 8}{x + 3}$.

9. **Linear Approximation and Tangent Lines**

(a) Estimate $\sqrt[4]{15}$ using a linear approximation of the function $\sqrt[4]{x}$ at the point 16.

(b) Find an equation of the line tangent to $y = \frac{x+1}{x-1}$ at the point $x = 2$.