Math 1231 Fall 2020
Single-Variable Calculus I Mastery Quiz 2
Due Noon on Tuesday, September 15

This week's mastery quiz has three topics. Please answer the problems on the new topics, labeled 3 and 2. You may answer topic 1 if you did not get a mastery on it last week.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. You shouldn't spend more than 10-20 minutes on this quiz. Feel free to consult your notes, but please don't talk about the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please upload your work as *one PDF file*. You can produce the file on your computer/tablet/whatever, or you can handwrite it and then scan it. If you have a smartphone, there are many apps that can help you produce a clean single pdf; I personally have used GeniusScan but there are many options.

Topics:

3. Computing Limits

2. Formal limits

1. Informal limits and continuity

3. **Computing Limits** Compute:

(a) $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3} =$

Solution:

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} 1\sqrt{x+1}+2 = \frac{1}{4}$$

(b) $\lim_{x\to 2} \dfrac{x^2+x-5}{3-x} =$

Solution:

$$\lim_{x\to 2} \frac{x^2+x-5}{3-x} = \frac{1}{1} = 1.$$

(c) $\lim_{x\to 1} \dfrac{1}{x-1} - \dfrac{1}{x^2-x} =$

Solution:

$$\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x\to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x\to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x\to 1} \frac{1}{x} = 1.$$

2. **Formal Limits**

(a) Write a formal ϵ-δ proof that $\lim_{x\to 2} 3x+1 = 7$.

Solution: Let $\epsilon > 0$ and set $\delta = \epsilon/3$. Then if $0 < |x-2| < \delta$, we have

$$|3x+1-7| = |3x-6| = 3|x-2| < 3\delta = \epsilon.$$

(b) Explicitly naming each limit law you use, compute

$\lim_{x\to 2} 3 \cdot \dfrac{x^2-4}{x-2} =$

Solution:

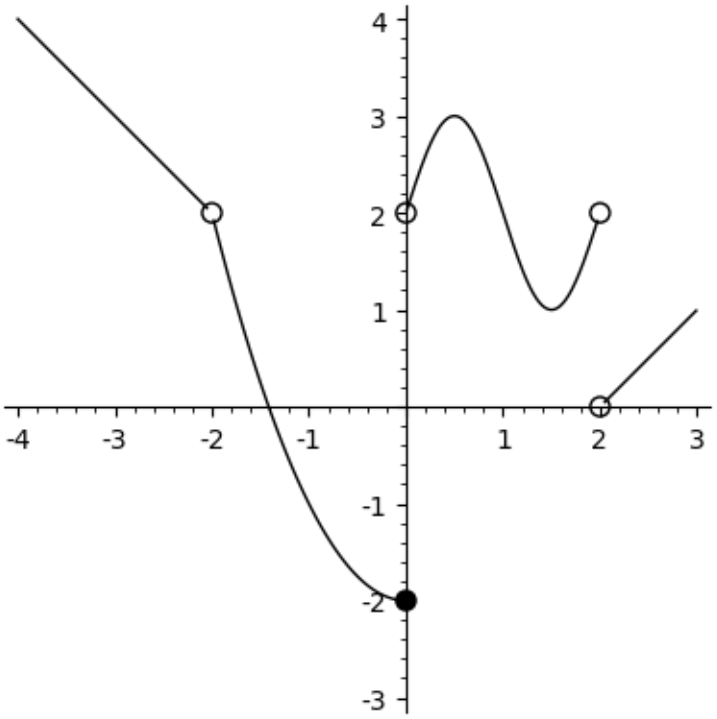
$$\begin{aligned}
\lim_{x\to 2} 3 \cdot \frac{x^2-4}{x-2} &= \lim_{x\to 2} 3 \cdot \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} && \text{Products} \\
&= \lim_{x\to 2} 3 \cdot \lim_{x\to 2} (x+2) && \text{Almost Identical Functions} \\
&= \lim_{x\to 2} 3 \left(\lim_{x\to 2}(x) + \lim_{x\to 2}(2) \right) && \text{additivity} \\
&= 3 \left(\lim_{x\to 2} x + 2 \right) && \text{constants} \\
&= 3(2+2) = 12 && \text{identity.}
\end{aligned}$$

1. **Informal Continuity and Limits**

(a) Give an approximate value for sin(.1), and explain how you got it.

Solution: We know that .1 is close to 0, so we expect sin(.1) to be close to $\sin(0) = 0$.

Here is the graph of a function f:



For each of the following questions, if your answer is "does not exist", explain in a few words why it does not exist. If your answer is just a number, you don't need to explain.

(b) What is the domain of f?

Solution: All reals except -2 and 2. I'd also except all of $[-4, 3]$ except -2 and 2.

(c) Where (if anywhere) is f discontinuous?

Solution: $x = -2, 0, 2$.

(d) What is $\lim_{x \to -2} f(x)$?

Solution: 2

(e) What is $f(-2)$?

Solution: Does Not Exist, since the function is undefined at -2

(f) What is $\lim_{x \to 0} f(x)$?

Solution: Does Not Exist, since the limits are different from each side; this is a jump discontinuity.

(g) What is $f(0)$?

Solution: -2